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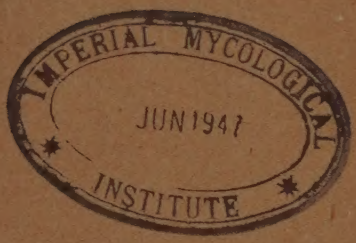

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MATHÉMATIQUES

SUR LA COMPACITÉ DES SYSTÈMES DE FONCTIONS PRESQUE PÉRIODIQUES GÉNÉRALISÉES DE H. WEYL

Par A. S. KOVANKO

(Présenté par A. N. Kolmogoroff, de l'Académie, le 28.X.1943)

Le but de la présente Note est de trouver les conditions pour qu'un système de fonctions presque-périodiques généralisées de H. Weyl (1) soit compact. Pour traiter cette question nous faisons usage des résultats dus à M. Lusternik (2) concernant les systèmes compacts de fonctions presque-périodiques de H. Bohr, ainsi que de nos résultats (3) sur les fonctions de H. Weyl et des résultats de M. Besicovitch (4) sur ses généralisations des fonctions de H. Bohr. Introduisons les notations et les formules suivantes.

I. Soit $E(a, b)$ la partie d'un ensemble mesurable E situé sur l'intervalle (a, b). $|E(a, b)|$ désigne sa mesure. Posons

$$\delta_w E = \lim_{T\to\infty}\left[\sup_{-\infty<a+<\infty}\frac{|E(a-T, a+T)|}{2T}\right]$$

II. Soit $f(x)$ et $\varphi(x)$ deux fonctions mesurables sur $(-\infty, +\infty)$. Soit $E_A(\subset E)$ l'ensemble de points, où $|f(x)-\varphi(x)|\geqslant A$ sur E. Introduisons la distance entre $f(x)$ et $\varphi(x)$ dans l'espace des fonctions mesurables $\delta_w^E(f, \varphi)$

$$\delta_w^E(f, \varphi) = \inf_{0\leqslant A\leqslant\infty}[A+\delta_w E_A]$$

Si $E=(-\infty, +\infty)$, alors nous écrivons simplement $\delta_w(f, \varphi)$.

III. Soit $f(x)$ et $\varphi(x)$ deux fonctions sur $(-\infty, +\infty)$ telles que $|f(x)|^\omega$ et $|\varphi(x)|^\omega$ $(\omega\geqslant 1)$ sont sommables. Nous introduisons la distance $D_{w_\omega}^E(f, \varphi)$

$$D_{w_\omega}^E(f, \varphi) = \lim_{T\to\infty}\left\{\sup_{-\infty<a<+\infty}\left[\frac{1}{2T}\int_{E(a-T,\ a+T)}|f-\varphi|^\omega dx\right]^{1/\omega}\right\}$$

Si $E=(-\infty, +\infty)$, nous écrivons simplement $D_{w_\omega}(f, \varphi)$.

IV. Soit $[f]_N = f$, si $|f|<N$, et $[f]_N = N\frac{f}{|f|}$, si $|f|\geqslant N$.

Définition I (3). La fonction $f(x)$ $(-\infty\leqslant x\leqslant+\infty)$ est dite $\tilde{w}$ presque-périodique ($\tilde{w}$ p. p.) si, quel que soit $\varepsilon>0$, il existe un ensemble relativement dense* de nombres $\{\tau\}$ (presque-périodes) tels que

$$\delta_w(f(x+\tau),\ f(x))<\varepsilon$$

Définition II (1). La fonction $f(x)$ $(-\infty<x<+\infty)$ est dite w_ω presque-périodique (w_ω p. p.) si, quel que soit $\varepsilon>0$, il existe un ensemble relativement dense de nombres $\{t\}$ (presque-périodes) tels que

$$D_{w_\omega}(f(x+\tau),\ f(x))<\varepsilon$$

* Un ensemble E est dit relativement dense, s'il existe un nombre $a>0$ fixe tel que l'ensemble E $(x,\ x+a)$ est non vide, quel que soit x.

Théorème 1. *Les espaces métriques, définis par les distances* $\mathfrak{d}_w^E$ *et* $D_{w_\omega}^E$ *sont des espaces non complets. En particulier, les espaces de fonctions presque-périodiques de H. Bohr, définis resp. par les distances* $\mathfrak{d}_w^E$ *et* $D_{w_\omega}^E$, *sont des espaces non complets.*

Pour éviter toute confusion dans la question de la compacité des systèmes de fonctions $\tilde{w}$ p. p. et w_ω p. p., nous avons à introduire des fonctions idéales limites pour compléter notre espace non complet.

Soit $x_1, x_2, x_3, \ldots$ une suite de points d'un espace non complet telle que $\lim_{n\to\infty} |x_{n+p} - x_n| = 0$* et qui n'a pas de point limite. Nous introduisons un point limite idéal $\overset{(i)}{x}$ pour cette suite.

Soit une autre suite $y_1, y_2, y_3, \ldots$ qui converge vers un point limite idéal $\overset{(i)}{y}$. Nous introduisons la notion de distance entre deux points idéals $\overset{(i)}{x}$ et $\overset{(i)}{y}$ comme il suit:

$$|\overset{(i)}{x} - \overset{(i)}{y}| = \lim_{\substack{m\to\infty \\ n\to\infty}} |x_m - y_n|$$

Théorème 2. *La condition nécessaire et suffisante de la compacité d'un système* $\mathfrak{M}(f)$ *de fonctions* $\tilde{w}$ p. p. *au sens de la distance* $\mathfrak{d}_w$ *consiste en ce que, quel que soit* $\varepsilon > 0$:

1) *il existe un nombre* $N_0 > 0$ *tel que* $\mathfrak{d}_w |(f, [f]_N) < \varepsilon$ *pour* $N > N_0$, *et cela pour toute fonction* $f(x)$ *du système* $\mathfrak{M}(f)$;

2) *il existe un nombre* $\eta > 0$ *tel que* $\mathfrak{d}_w (f(x+h), f(x)) < \varepsilon$ *pour* $|h| < \eta$, *et cela pour toute fonction* $f(x)$ *du système* $\mathfrak{M}(f)$;

3) *il existe un ensemble relativement dense de presque-périodes* $\{\tau\}$ *tels que* $\mathfrak{d}(f(x+\tau), f(x)) < \varepsilon$ *pour toutes les fonctions du système* $\mathfrak{M}(f)$.

Théorème 3. *La condition nécessaire et suffisante de la compacité d'un système* $\mathfrak{M}(f)$ *de fonctions* w_ω p. p. $(\omega \geqslant 1)$ *au sens de la distance* D_{w_ω} *consiste en ce que, quel que soit* $\varepsilon > 0$:

1) *il existe un nombre* $\sigma > 0$ *tel que* $D_{w_\omega}^E (f, 0) < \varepsilon$ *pour* $\delta_w E < \sigma$, *et cela pour toute fonction* $f(x)$ *du système* $\mathfrak{M}(f)$;

2) *il existe un nombre* $\eta > 0$ *tel que* $D_{w_\omega} (f(x+h), f(x)) < \varepsilon$ *pour* $|h| < \eta$, *et cela pour toute fonction* $f(x)$ *du système* $\mathfrak{M}(f)$;

3) *il existe un ensemble relativement dense de presque-périodes* $\{\tau\}$ *tels que* $D_{w_\omega} (f(x+\tau), f(x)) < \varepsilon$ *pour toutes les fonctions* $f(x)$ *du système* $\mathfrak{M}(f)$.

Définition III. $\left.\begin{matrix} a \\ b \end{matrix}\right\}$. La fonction $f(x)\ (-\infty < x < +\infty)$ est dite $\left.\begin{matrix} a \\ b \end{matrix}\right\} \begin{matrix} \tilde{w} \\ w_\omega \end{matrix}$ normale, si le système de tous les déplacements de cette fonction $\mathfrak{M}(f(x+k))$ est compact au sens de la distance $\left.\begin{matrix} a \\ b \end{matrix}\right\} \begin{matrix} \mathfrak{d}_w \\ D_{w_\omega} \end{matrix}$.

Théorème 4. $\left.\begin{matrix} a \\ b \end{matrix}\right\}$. *La classe des fonctions* $\left.\begin{matrix} a \\ b \end{matrix}\right\} \begin{matrix} \tilde{w} \\ w_\omega \end{matrix}$ *normales se confond avec celle de fonctions* $\left.\begin{matrix} a \\ b \end{matrix}\right\} \begin{matrix} \tilde{w} \text{ p.p.} \\ w_\omega \text{ p.p.} \end{matrix}$

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* $(x—y)$ désigne la distance entre x et y.

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HEAT ENGINEERING

HYDRODYNAMICAL THEORY OF HEAT EXCHANGE AND DIFFUSION IN TURBULENT GAS-FLOW IN PIPES

By Z. F. CHUKHANOV, Corresponding Member of the Academy

In 1874 Reynolds (1) published a paper in which he drew a parallel between friction and heat exchange in a flow; this idea was placed at the basis of the hydrodynamical theory of heat exchange. The interest in this theory greatly increased after the appearance of the remarkable papers by Tylor (2), Prandtl (3) and Karman (4).

Reynolds' equation for gases can be written down in the form:

$$q = \tau_0 \frac{c_p g (t - t_w)}{\bar{w}} \tag{1a}$$

or in the following form:

$$\frac{\mathrm{Nu}}{\mathrm{Pe}} = \frac{\zeta}{8} \tag{1b}$$

Here q is quantity of heat given out by the flow; τ_0, tangent stress on the wall; c_p, specific heat of the gas; $\bar{w}$, mean velocity of the flow; t, mean temperature of the flow; t_w, temperature of the wall; $\mathrm{Nu} = \alpha d/\lambda$, Nusselt's number; $\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{\bar{w}d}{\nu} \cdot \frac{\nu}{a} = \frac{\bar{w}d}{a}$, Peclet's number; ζ, coefficient of resistance; α, total coefficient of heat exchange; λ, coefficient of thermal conductivity; d, diameter of the pipe; ν, kinematic viscosity; a, thermometric conductivity.

Prandtl represented the scheme of turbulent flow as follows: the flow consists of two regions, the turbulent core with typical fluctuations of velocity, and a stationary boundary laminar layer where the fluid moves in parallel streamlines.

While applying Reynolds' analogy to the turbulent core and taking account of the phenomena observed in the boundary layer, Tylor and Prandtl gave the following equation for the heat exchange in compressible and incompressible fluids:

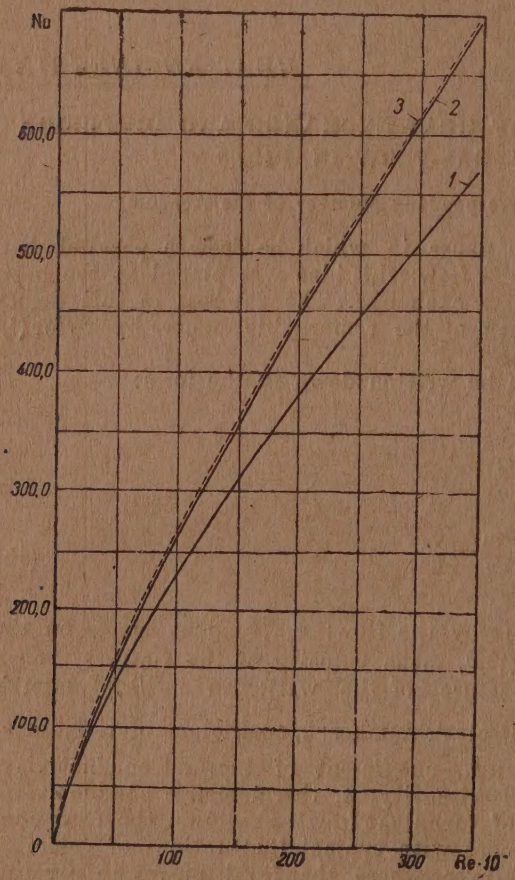
$$\frac{\mathrm{Nu}}{\mathrm{Pe}} = \frac{\zeta}{8} \cdot \frac{1}{1 + \frac{w_1}{\bar{w}} (\mathrm{Pr} - 1)} \tag{2a}$$

$$\frac{w_1}{\bar{w}} = C\,\mathrm{Re}^{-1/8} \tag{2b}$$

Karman refined the solution of the problem by studying the «intermediary region» between the boundary layer and the turbulent core, and arrived at a more complicated equation:

$$\frac{Nu}{Pe} = \frac{\zeta}{8} \frac{1}{1 + Re^{-1/8}\{Pr - 1 + \lg[1 + 5/6(Pr - 1)]\}} \quad (3)$$

Though it has made a considerable progress, the hydrodynamical theory of heat exchange is still far from being perfect. Even in the case of heat exchange in gas ($Pr \cong 1$) —which is the most simple case, inded—the experimental data deviate considerably from the theoretical, especially in the region of large values of Re.



Heat exchange in the turbulent gas flow in a tube: *1*, according to theoretical equation of Reynolds, Tylor, Prandtl and Carman ($Nu = 0.0396\ Re^{0.75}$); *2*, according to the empirical equation of Datus and Belter ($Nu = 0.0254\ Re^{0.8}$); *3*, according to the theoretical equation of the author

$$\left(Nu = 0.0793 \frac{Re^{0.75}}{1 + \frac{w_1}{w}\left(2 - \frac{w_1}{w}\right)}, \quad \frac{w_1}{w} = CRe^{-1/8}\right)$$

This circumstance is brought out by the adjoned figure in which the theoretical curve $Nu \rightarrow f(Re)$ obeying equation (1) [for $Pr = 1$ the equations of Prandtl and Karman coincide with equation (1)] is compared with the empiric curve (4) obtained from numerous experimental investigations and expressed analytically for the gas by the following relation

$$Nu = 0.0254\ Re^{0.8} \quad (4)$$

This figure shows plainly how strong is the divergence between the theoretical curve and the experimental data.

Usually this divergence is ascribed to the assumptions which are put at the basis of the hydrodynamical theory of heat exchange. It will be obvious, however,

that no fruitful development of the hydrodynamical theory or its extension to the heat exchange in liquids for $Pr \neq 1$ is possible, unless a satisfactory solution conformable to the experiment will be found for the simplest case of heat exchange in a gas flow.

The way towards the solution of this problem seems to lie in the investigation of the mechanism of the interaction between the turbulent flow and the surface of heating. The present author has analysed the ingenious experiments of Fage and Townend ([5]) in which the turbulent flow* was studied without any such devices being introduced as might distort it to a certain extent. On the basis of this analysis I have made an attempt to build a scheme of the mechanism of interaction between the turbulent flow and the surface of the pipe walls, and from it to derive the equations of the hydrodynamical theory of heat exchange. This has not been an unsuccessful attempt, as may be seen by comparing the author's equation with the empirical curve; the curve built according to this equation practically coincides with the empirical curve within a wide range of variation of Re.

We shall not dwell here on the analysis of Fage and Townend's results which has led to the scheme of the flow mechanism as reported here.

Under a turbulent regime there is no rectilinear motion of the fluid, and what is usually meant by a stationary laminar boundary sublayer is absent. Because of the damping of transverse pulsations, the conditions of motion near the wall are sharply different from those in the core of the flow. Near the wall the longitudinal pulsations are comparatively increased, and the conditions created here bear a very little resemblance to what is ordinarily described as laminar regime.

Nor are they similar to the normal turbulent regime of the core. Considering a certain point near the surface of the wall, we may state, in the first place, that the motion of the fluid at this point and near it is not a stationary process. In fact, the appearance of a velocity component perpendicular to the wall and the obviously turbulent character of the motion in this region alternates with the absence of a transverse pulsation when the flow assumes certain features of a laminar regime.

A characteristic feature of this pseudolaminar regime is that the transverse migration of matter and the transfer of heat are effected solely by way of diffusion and conduction, that is, in the molecular way. Instantaneous photography of the process taking place near the wall reveals an undulatory surface which at this instant is the quenching boundary of transverse velocity pulsations**. This boundary being variable with time, the thickness of the pseudolaminar boundary layer and the quantities characterizing the interface and the layer itself may only be spoken of as mean statistical values.

The interaction between the turbulent core and the boundary layer is determined by the mutual exchange of fluid due to velocity pulsations in the direction parallel to the wall. Obviously, this exchange is possible only because the interface—the border of the laminar boundary layer—is not parallel to the wall of the pipe. It is also natural that the resistance to the motion of the turbulent flow, as well as the quantities of heat and matter transferred to the boundary layer will be determined by the exchange of matter between the turbulent flow and the pseudolaminar layer, and also by the conditions under which the corresponding process in the boundary layer takes place.

On the basis of the above scheme suggested for the mechanism of turbulent flow a quantitative relation may be established between the resistance and the heat exchange, and also between the resistance and the exchange of matter by diffusion in the presence of a concentration gradient towards the wall of the pipe.

If we suppose as usual that τ in the boundary layer is constant and equal to τ_0, that is the total resistance to the movement of the fluid is practically equal to the

* Our scheme of turbulent flow is confirmed also by the work of L. S. Eigenson on the fluctuations of pressure and temperature on the wall of the pipe.

** Naturally enough, our scheme is imperfect in many respects, and its detailed elaboration requires a careful experimental investigation of the pseudolaminar boundary layer.

resistance to the movement of the turbulent core, then the loss of head will, according to our scheme, be determined by the loss of kinetic energy of that portion of the fluid in the turbulent core which penetrates into the pseudolaminar layer. Denote by Δ the amount of fluid* penetrating in unit time into the boundary layer on a dx long portion of the pipe; by dp, the head-loss in this portion; and by r, the radius of the pipe. The conditions of equilibrium may then be written as follows:

$$-\pi r^2 dp = \tau_0 2\pi r\, dx \tag{5}$$

As

$$dp = \frac{\Delta}{\pi r^2 \bar{w}} \frac{\bar{w}^2 - w_1^2}{2g} \tag{6}$$

we have

$$\tau_0 = \frac{\Delta}{2\pi r\, dx} \frac{\bar{w}^2 - w_1^2}{2\bar{w}g} \tag{7}$$

Since the transfer of heat from the turbulent core is effected by the same volume of the fluid, the amount of heat Q transferred to the pseudolaminar layer is determined by the equation

$$Q = \Delta c_p (\bar{t} - t_1) \tag{8}$$

Or, per unit area of the wall surface

$$q = \frac{\Delta c_p (\bar{t} - t_1)}{2\pi r\, dx} \tag{9}$$

After Δ is found from the equation (7) and substituted in (9), one obtains the equation defining the quantity of heat transferred from the turbulent core to the boundary layer

$$q = \tau_0 \frac{c_p g (\bar{t} - t_1)}{\bar{w} - w_1} \cdot \frac{2\bar{w}}{\bar{w} + w_1} \tag{10}$$

By means of simple transformations and adopting the notation

$$\theta = \frac{\bar{t} - t_1}{\bar{t} - t_w} \tag{11}$$

equation (10) can be reduced to a form analogous to equation (1b)

$$\frac{\mathrm{Nu}}{\mathrm{Pe}} = \frac{\zeta}{8} \frac{2\theta}{1 - \left(\frac{w_1}{\bar{w}}\right)^2} \tag{12}$$

For determining θ let us turn to the processes taking place in the pseudolaminar boundary layer.

The condition $\tau = \tau_0$ corresponds to a linear distribution of velocity in the boundary layer, *i. e.*

$$\tau_0 = \mu \frac{w_1}{\delta} \tag{13}$$

where μ is the viscosity of the fluid, and δ, the mean statistical thickness of the boundary layer.

The amount of heat transferred to the wall of the pipe is determined, if one takes account of the equation (13), by

$$q = \frac{\lambda \tau_0}{\mu w_1} (t_1 - t_w) \tag{14}$$

* Naturally, Δ refers to a gas having a velocity $\bar{w}$.

From (10) and (14) we find the value of θ:

$$\theta = \frac{1 - \left(\frac{\overline{w_1}}{\overline{w}}\right)^2}{1 + \frac{\overline{w_1}}{\overline{w}}\left(2\mathrm{Pr} - \frac{\overline{w_1}}{\overline{w}}\right)} \tag{15}$$

Substituting in (12) the value of θ from (15), we ultimately obtain

$$\frac{\mathrm{Nu}}{\mathrm{Pe}} = \frac{\zeta}{4} \frac{1}{1 + \frac{\overline{w_1}}{\overline{w}}\left(2\mathrm{Pr} - \frac{\overline{w_1}}{\overline{w}}\right)} \tag{16}$$

or, in a form analogous to (10),

$$q = \tau_0 \frac{c_p g\,(t - t_w)}{\overline{w}} \frac{2}{1 + \frac{\overline{w_1}}{\overline{w}}\left(2\mathrm{Pr} - \frac{\overline{w_1}}{\overline{w}}\right)} \tag{17}$$

The equation (16) which determines the heat exchange in the turbulent flow is valid for gases with $\mathrm{Pr} \cong 1$. The application of the equation (16) in practice implies the determination of ζ and of the ratio $\frac{\overline{w_1}}{\overline{w}}$ typical of the flow; ζ is determined solely by the loss of head connected with the interaction between the turbulent core and the pseudolaminar boundary layer.

For a smooth walled pipe the loss of head is practically the total loss of head. Analytically, ζ for $\mathrm{Re} \leqslant 250000$ is rather accurately described by the well-known equation of Blasius

$$\zeta = 0.3164\ \mathrm{Re}^{-1/4} \tag{18}$$

If necessary, ζ can be determined from other equations semi-theoretical or empirical [cf. ([6-7])]. More complicated is, of course, the determination of ζ for rough surfaces, for channels of complicated form, in the case of external flow past a body, when considerable losses of head take place, not connected with the interaction between the turbulent core and the boundary layers. This question will not be entered into here.

More complicated in principle is the determination of the characteristic ratio $\frac{\overline{w_1}}{\overline{w}} = W$, because, being a mean statistical quantity, it has so far defied direct experimental determination.

There is no doubt that W is a function of the physical properties of the fluid and of the character of the flow, *i. e.* in our case of the value Re.

To put $W = \mathrm{const}$, as is sometimes done, is obviously unlawful, since it is just the variation of this quantity that characterizes the variation of the hydrodynamical conditions under which the process develops and goes on. But W may hardly be supposed to be a function of Pr either, since by the very nature of the process W will be influenced simply by the relative viscosity of the fluid in the boundary layer, determining the relation of the velocity gradient in the boundary layer and in the turbulent core. We shall consider this problem in more detail while analysing the heat exchange in liquids. This paper is confined to the turbulent flow in gases, and we must obtain for W a simple relation in the form

$$W \rightarrow f(\mathrm{Re}) \tag{19}$$

for in the case of compressible fluid one may assume roughly that the velocity gradient in the turbulent core is equal to that in the boundary layer.

As it is the averaged values of $\overline{W}$ and $\overline{\delta}$ that are considered, $\overline{W}$ for a gas can within the applicability range of the Blasius law be determined according to Prandtl from the equation (2b), where C_1 lie within 1.8—2.48.

Taking into account the relations (2b) and (18), the equation of the heat exchange for a gas can be written down in the following form:

$$Nu = \frac{a}{4} \frac{Re^{0.75}}{1+2C\,Re^{-1/8}-C^2\,Re^{-1/4}} \qquad (20)$$

The run of the curve according to equation (20) depends upon the value of the coefficient which appears to lie nearer to 1.8, and on certain considerations may be approximately put at 2.0*.

A curve built according to equation (20) for $C_1=2.0$ is given in the figure. The variation of C up to 1.8 does not significantly affect the run of the curve. It may be seen from the figure that there is a very satisfactory agreement between the theoretical curve and the empirical curve obtained as a generalization of a vast experimental material.

It may easily be shown that the equation (16) is also applicable to the solution of the problem of diffusion to the wall of a channel, *e. g.* in the case of burning of coal channels and in other heterogeneous processes taking place on the walls of a channel.

Naturally enough, α_{diff} must be taken instead of α, Nu_{diff} instead of Nu, and so on; Nu_{diff} as well as Nu are determined as functions of Re, from the figure.

In conclusion we are to make a remark upon the influence of the direction of the thermal flow upon its magnitude.

Equation (16) allows for the influence of the direction of the thermal flow, since by the very essence of the process the constants entering into the various quantities refer to a definite region of the flow. ζ as well as W are determined in the boundary layer or approximately ν_w, whereas the thermometric conductivity in Pe is determined for the conditions prevailing in the turbulent core.

Lenin Institute of Railway Engineers.
Tbilissi.

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* From the known relation $\frac{w_1}{\sqrt{\frac{\tau}{\rho}}} = \text{const}$ we obtain $\frac{w_1}{w} = A\sqrt{\xi}$ and for C_1 the value ~2, more accurately, 2.07.

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ELECTRICAL ENGINEERING

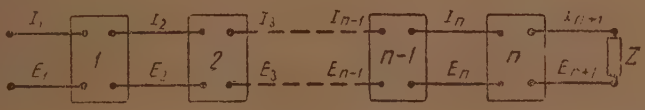
ON REPEATED RESISTANCE OF SYMMETRICAL UNIFORM CHAIN SCHEMES

By P. L. KALANTAROV and L. A. ZEITLIN

(Communicated by V. F. Mitkevich, Member of the Academy, 15.I.1944)

1. Uniform chain schemes, that is such as consist of similar links, are widely used in wire communication engineering, especially in the capacity of filters. For the most part the links, and, accordingly, the entire scheme, are symmetrical with respect to the leading-in and leading-out terminals. Such link and the scheme as a whole present four-poles.

One of the essential characteristics of a four-pole is the repeating resistance which means that, having closed the leading-out terminals of the four-pole through such a resistance, one obtains an identical resistance between the leading-in terminals. Considering a uniform chain scheme whose leading-out terminals are closed through a resistance equal to the repeating resistance of the link, we readily come to realize that this resistance is also the repeating resistance of the whole scheme. However, as will be shown later, the repeating resistance of the whole scheme can in addition take values different from the repeating resistance of the link.



2. If a chain scheme consisting of similar symmetrical links (see figure) be considered as a four-pole, one can write

$$\dot{E}_1 = A_n\dot{E}_{n+1} + B_n\dot{I}_{n+1}; \quad \dot{I}_1 = C_n\dot{E}_{n+1} + A_n\dot{I}_{n+1} \qquad (1)$$

where A_n, B_n, C_n are complex constants of a symmetrical four-pole bound up by the relation $A_n^2 - B_nC_n = 1$. For the part of the scheme formed by the first $(n-1)$ links and for the last link we have

$$\dot{E}_1 = A_{n-1}\dot{E}_n + B_{n-1}\dot{I}_n; \qquad \dot{I}_1 = C_{n-1}\dot{E}_n + A_{n-1}\dot{I}_n \qquad (2)$$

and

$$\dot{E}_n = A_1\dot{E}_{n+1} + B_1\dot{I}_{n+1}; \qquad \dot{I}_n = C_1\dot{E}_{n+1} + A_1\dot{I}_{n+1} \qquad (3)$$

respectively, where A_1, B_1, C_1 are complex constants of a link, connected by the relation $A_1^2 - B_1C_1 = 1$.

From (2) and (3) we find

$$\dot{E}_1 = (A_1A_{n-1} + C_1B_{n-1})\dot{E}_{n+1} + (A_1B_{n-1} + B_1A_{n-1})\dot{I}_{n+1}$$
$$\dot{I}_1 = (A_1C_{n-1} + C_1A_{n-1})\dot{E}_{n+1} + (A_1A_{n-1} + B_1C_{n-1})\dot{I}_{n+1}$$

Hence

$$A_n = A_1A_{n-1} + C_1B_{n-1} = A_1A_{n-1} + B_1C_{n-1} \quad (4)$$
$$B_n = A_1B_{n-1} + B_1A_{n-1} \quad (5)$$
$$C_n = A_1C_{n-1} + C_1A_{n-1} \quad (6)$$

Putting in (4), (5) and (6) $n = 2, 3, 4, \ldots$ and making use of the relation $A_1^2 - B_1C_1 = 1$, we obtain

$$B_n = B_1F_n(a); \quad C_n = C_1F_n(a) \quad (7)$$

where $a = 2A_1$, and the shape of the function $F_n(a)$ varies with the number of links in the scheme. In the second column of the table presented below the function $F_n(a)$ is given for n ranging from 2 to 8.

n	$F_n(a)$	Values of $A_1 = a/2$ for which $F_n(a) = 0$
2	a	0
3	$a^2 - 1$	± 0.500
4	$a^3 - 2a$	± 0.707 0
5	$a^4 - 3a^2 + 1$	± 0.809 ± 0.309
6	$a^5 - 4a^3 + 3a$	± 0.866 ± 0.500 0
7	$a^6 - 5a^4 + 6a^2 - 1$	± 0.901 ± 0.625 ± 0.223
8	$a^7 - 6a^5 + 10a^3 - 4a$	± 0.924 ± 0.707 ± 0.383 0

It may easily be seen that for $F_n(a)$ the expression

$$F_n(a) = a^{n-1} - \frac{n-2}{1!}a^{n-3} + \frac{(n-3)(n-4)}{2!}a^{n-5} - \frac{(n-4)(n-5)(n-6)}{3!}a^{n-7} + \ldots$$

is valid, if we agree upon omitting the terms with negative powers of a.

Note also that from (7) follows the relation

$$\frac{B_n}{C_n} = \frac{B_1}{C_1} \quad (8)$$

3. Suppose now that the leading-out terminals of our chain scheme are closed for resistance Z. Then $\dot{E}_{n+1} = Z\dot{I}_{n+1}$ and from (1) we have

$$\frac{\dot{E}}{\dot{I}_1} = \frac{ZA_n + B_n}{ZC_n + A_n} = Z\frac{A_n + \frac{B_n}{Z}}{A_n + ZC_n}$$

It will be obvious that for $\frac{B_n}{Z} = ZC_n$ the terminal resistance Z is the repeating resistance of the scheme.

The above relation takes place if

$$Z = \sqrt{\frac{B_n}{C_n}} \quad (9)$$

or else if

$$B_n = 0 \quad \text{and} \quad C_n = 0 \quad (10)$$

For a link these conditions are

$$Z = \sqrt{\frac{B_1}{C_1}} \qquad (11)$$

$$B_1 = 0 \quad \text{and} \quad C_1 = 0 \qquad (12)$$

Below, the conditions (9) and (11) are referred to as conditions of the first kind, and those (10) and (12) as second kind.

If follows from (8) that (9) and (11) are the same for a simple link and for the whole scheme. These conditions are generally known, and it will suffice here to point out the fact that they are independent of the number of links.

But the existence of the conditions of the second kind does not appear to have ever been called to attention.

4. Let us therefore discuss these conditions in greater detail. It follows from (7) that if the conditions of the second kind are fulfilled for a single link, they also hold good for the chain scheme as a whole. But it may be seen from the same relations (7) that the conditions of the second kind for the whole scheme (10) can be satisfied even if the respective conditions for a link (12) are not satisfied, and we have

$$F_n(a) = 0 \qquad (13)$$

Thus, unlike the conditions of the first kind, those of the second kind depend upon the number of links but not upon the value of terminal resistance of the scheme, and if they are fulfilled, any terminal resistance is a repeating resistance.

To sum up, the terminal resistance of a chain scheme is at once its repeating resistance if one of the following conditions is satisfied:

1) $\frac{B_n}{C_n} = \frac{B_1}{C_1} = Z^2$ (first kind)

2) $B_1 = 0, \quad C_1 = 0$ } (second kind)

3) $F_n(a) = 0$ }

5. The results of this work have been applied by the authors to determining the maxima of the resonance curve in the case of symmetrical uniform chain schemes.

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PHYSICS

A NEW EXACT SOLUTION OF NAVIER-STOKES EQUATIONS

By L. LANDAU

(Communicated by P. L. Kapitza, Member of the Academy, 25.I.1944)

The problem of fluid motion in a laminar jet discharged into a still fluid has been solved by Schlichting for large Reynolds numbers. In the present work the author shows that the motion in an axially symmetrical jet spreading in an unbounded space can be determined accurately for arbitrary Reynolds numbers by a rigorous solution of Navier-Stokes equations.

Let us consider a jet discharged, say, from a thin pipe into an unbounded space. We choose spherical coordinates r, θ, φ with polar axis directed along the velocity of the jet at its exit point, which we define to be located at the origin. The motion is axially symmetrical around the polar axis, so that $v_\varphi = 0$ and v_r, v_θ are functions of r and θ only. Any closed surface around the origin (including the particular case of a surface removed to infinity) should be traversed by the same full flow of momentum («momentum of the jet»). This requires that the velocity should vary inversely as the distance r from the origin, so that

$$v_r = \frac{1}{r} F(\theta), \qquad v_\theta = \frac{1}{r} f(\theta) \tag{1}$$

where F, f are functions of θ only. The continuity equation runs thus

$$\frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta \cdot v_\theta) = 0$$

By means of it we find that

$$F(\theta) = -\frac{df}{d\theta} - f \operatorname{ctg} \theta \tag{2}$$

We introduce the tensor Π_{ik} of the density of the momentum flow according to

$$\Pi_{ik} = p\delta_{ik} + \rho v_i v_k - \eta \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right)$$

It permits the equation of motion of an incompressible viscous liquid (Navier-Stokes equations) to be written in the form

$$\sum_{k=1}^{3} \frac{\partial \Pi_{ik}}{\partial x_k} = 0$$

In spherical coordinates the components $\Pi_{r\varphi}$, $\Pi_{\theta\varphi}$ of the tensor of momentum flow density in the jet vanish identically, as is plain enough from considerations of symmetry. Let us make the assumption that the components $\Pi_{\theta\theta}$ and $\Pi_{\varphi\varphi}$ are likewise zero (the justification for it lies in the fact that we thereby obtain a solution satisfying all necessary conditions). With the aid of the well-known expressions for the components $\frac{\partial v_i}{\partial}{}_k + \frac{\partial v_k}{\partial x_i}$ in spherical coordinates and of the formulae (1)—(2) one easily realizes that the following relation connects the components $\Pi_{\theta\theta}$, $\Pi_{\varphi\varphi}$ and $\Pi_{r\theta}$ of the tensor Π_{ik} in the jet:

$$\sin^2\theta \cdot \Pi_{r\theta} = \frac{1}{r}\frac{\partial}{\partial\theta}\left[\sin^2\theta\,(\Pi_{\varphi\varphi} - \Pi_{\theta\theta})\right]$$

We have therefore to infer from $\Pi_{\varphi\varphi}$ and $\Pi_{\theta\theta}$, being equal to zero, that $\Pi_{r\theta} = 0$ also. Thus, of all of the components of Π_{ik} only Π_{rr}, which depends on both r and $1/r^2$, is distinct from zero. This being so, it may easily be seen that the equations of motion $\sum_k \partial\Pi_{ik}/\partial x_k = 0$ are satisfied automatically.

We then write

$$\frac{1}{\rho}(\Pi_{\theta\theta} - \Pi_{\varphi\varphi}) = \frac{1}{r^2}(f^2 + 2\nu f\operatorname{ctg}\theta - 2\nu f') = 0$$

$\left(\nu = \frac{\eta}{\rho}\right)$ or

$$\frac{d}{d\theta}\left(\frac{1}{f}\right) + \operatorname{ctg}\theta\,\frac{1}{f} + \frac{1}{2\nu} = 0$$

The solution of this equation is

$$f = -\frac{2\nu\sin\theta}{A - \cos\theta} \qquad (3)$$

(where A is constant), and from (2) we now get for F the expression

$$F = 2\nu\left\{\frac{A^2 - 1}{(A - \cos\theta)^2} - 1\right\} \qquad (4)$$

The distribution of pressure is determined from the equation

$$\frac{1}{\rho}\Pi_{\theta\theta} = \frac{p}{\rho} + \frac{f}{r^2}(f + 2\nu\operatorname{ctg}\theta) = 0$$

We obtain

$$p = -\frac{4\rho\nu^2}{r^2}\frac{(A\cos\theta - 1)}{(A - \cos\theta)^2} \qquad (5)$$

The constant A can be connected with the «momentum of the jet» P, that is with the full flow of momentum in it. This flow is equal to the integral over the spherical surface

$$P = \oint \Pi_{rr}\cos\theta\,df = 2\pi\int_0^{\pi} r^2\Pi_{rr}\cos\theta\sin\theta\,d\theta$$

The quantity Π_{rr} is equal to

$$\frac{1}{\rho}\Pi_{rr} = \frac{4\nu^2}{r^2}\left\{\frac{(A^2 - 1)^2}{(A - \cos\theta)^4} - \frac{A}{A - \cos\theta}\right\}$$

and the computation of the integral gives

$$P = 16\pi\nu^2\rho A\left\{1 + \frac{4}{3(A^2-1)} - \frac{A}{2}\ln\frac{A+1}{A-1}\right\} \tag{6}$$

Formulae (1)—(6) solve the problem.

The streamlines are determined by the equation $\frac{dr}{v_r} = \frac{r\,d\theta}{v_\theta}$; integrating it, one obtains

$$\frac{r\sin^2\theta}{A-\cos\theta} = \text{const} \tag{7}$$

Let us consider two limiting cases: a weak jet (momentum P small) and a strong jet (P large). With $P \to 0$ the constant A tends to infinity; from (6) we have

$$P = \frac{16\pi\nu^2\rho}{A} \tag{8}$$

For the velocity we obtain in this case

$$v_\theta = -\frac{P}{8\pi\nu\rho}\frac{\sin\theta}{r}; \qquad v_r = \frac{P}{4\pi\nu\rho}\frac{\cos\theta}{r} \tag{9}$$

With $P \to \infty$ (strong jet) A tends to one; here (6) gives

$$A = 1 + \frac{\alpha^2}{2} \quad \text{where} \quad \alpha = \frac{32\pi\nu^2\rho}{3P} \tag{10}$$

For large angles ($\theta \sim 1$) the velocity is determined by the formula

$$v_\theta = -2\nu\frac{\operatorname{ctg}\theta/2}{r}; \qquad v_r = -\frac{2\nu}{r} \tag{11}$$

For small angles ($\theta \sim \alpha$) we have

$$v_\theta = -\frac{4\nu\theta}{\alpha^2+\theta^2}; \qquad v_r = 8\nu\frac{\alpha^2}{(\alpha^2+\theta^2)^2} \tag{12}$$

in accordance with the results obtained by Schlichting (the rôle of the Reynolds number is played in this problem by the quantity $\frac{1}{\nu}\sqrt{\frac{P}{\rho}}$).

It may be remarked in conclusion that the solution as given here is rigorous for a jet supposed to be discharged from a point source. But if the finite dimensions of the orifice should be taken into account, the solution obtained would present the first term of the expansion in powers of the ratio between the dimensions of the orifice and the distance r from it. With this circumstance is connected the fact that the total flow of the fluid passing through a closed surface around the origin will be found, if calculated by means of the obtained solution, to be equal to zero, A total flow different from zero is obtained if the next terms of the above-mentioned expansion are taken into account.

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GEOPHYSICS

ON SCATTERING OF RADIATION BY CLOUDS

By N. N. KALITIN

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Up to the present time we lack any information as to the ability of a definite group of clouds to scatter the solar radiation falling upon them. Yet, such data are necessary both for investigating clouds from the standpoint of radiation, and for the procedure of approximate designs of radiational and light-regime for a definite point.

In order to compare clouds of different shape with respect to the flux of scattered radiation emitted by them, the following conditions are to be satisfied: 1) during the measurements the solar radiation must always fall upon the clouds at the same angle; 2) the cloud must occupy on the sky a constant place relative to the observer; 3) the insolated area of the cloud must not vary; 4) during the measurements the sky must be free of any other cloud-strata which may distort the results of the measurements by producing a screening effect upon the cloud under observation, or act upon it through the albedo of their surfaces.

All these conditions being strictly fulfilled, the present author studied in 1942—1943 at Vysokaja Dubrava, near Sverdlovsk, the scattering of radiation by clouds of various forms. Within this period a total of 1354 observation series could be obtained; the conclusions derived from them are discussed below. For the measurements use was made of Yanishevsky's pyranometer which was fixed in such a manner as to permit measurements of zenith radiation to be made at a visual angle of 40°. The measurements were carried out at an altitude of the Sun of 30 and 40° and in such cases only where in the zenith clouds of a single definite group were present, the sky being free of clouds of any other stage.

Observations were also carried out with cloudless sky in order to obtain reference data for the study of the radiation due to clouds.

The observational results are given in the table; only those shapes of clouds have been taken into account for which the number of observations exceeds twenty series: exception was made for *As*: with these clouds the number of observations with a Sun height of 40° was only thirteen.

Cirro-cumulus clouds (*Cc*) which have been observed ten times only, have not been included in the table. The following notation is adopted: A, relation between mean quantity of radiation with a cloud of a definite form and the quantity of radiation with cloudless sky; B, relation between the maximum quantity of radiation observed and the minimum one; C, relation between the medium quantity of radiation with a Sun altitude of 40° and the same quantity with a Sun altitude of 30°.

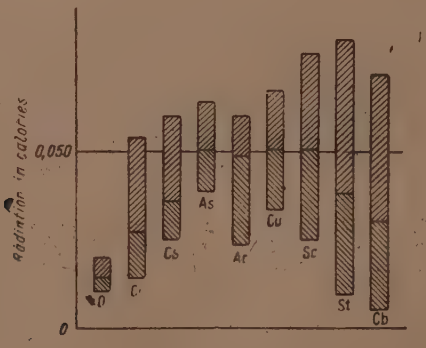
From the mean values it may be seen that the upper stage, represented by *Ci* and *Cs*, has the least scattering effect; the medium stage (*As*, *Ac*, *Sc*) gives much greater values, while the values given by the bottom stage *St* occupy an intermediate position.

Form of clouds	Sun altitude 30°						Sun altitude 40°						
	Number of observ.	Radiation, in cal. medium	maximum	minimum	*A*	*B*	Number of observ.	Radiation, in cal. medium	maximum	minimum	*A*	*B*	*C*
Without clouds	140	0.014	0.022	0.010	—	2.2	129	0.017	0.030	0.010	—	3.0	1.21
Ci	88	0.027	0.061	0.013	1.9	4.7	46	0.037	0.081	0.017	2.2	4.8	1.37
Cs	27	0.036	0.068	0.022	2.6	3.1	21	0.042	0.059	0.018	2.5	3.3	1.17
As	24	0.051	0.067	0.035	3.6	1.9	13	0.065	0.084	0.036	3.8	2.3	1.27
Ac	77	0.049	0.081	0.022	3.5	3.7	48	0.067	0.109	0.036	3.9	3.0	1.37
Cu	30	0.051	0.076	0.032	3.6	2.4	54	0.065	0.101	0.013	3.8	7.8	1.27
Sc	120	0.051	0.084	0.022	3.6	3.8	78	0.069	0.119	0.027	4.1	4.4	1.35
St	163	0.038	0.091	0.009	2.7	10.1	81	0.049	0.108	0.009	2.9	12.0	1.20
Cb	35	0.030	0.082	0.003	2.2	27.3	40	0.048	0.119	0.007	2.8	17.0	1.69

By the name of the lower stage, *St*, we understand here clouds of the following classes: *stratus*, *stratus-nimbus* and *nimbus*.

The clouds of ascending currents, *Cu*, approach in their radiation properties to those of the medium stage, while *Cb* approximate the bottom range of altitude.

Although the number of observations is not very large, the range of variation for definite forms of clouds can nevertheless be determined (relation between minimum and maximum values). From column *B* it may be seen that with respect to radiation the most stable are clouds of the group *As*, for which this relation is both with a Sun height of 30 and 40° equal to about 2. The most variable are the clouds of the group *Cb*, for which the value of *B* amounts to 27.



If a very large *Cb* cloud is situated at the zenith of the sky (clouds of this form are known to reach several km in size), it may absorb so much Sun radiation that quite insignificant amounts of the latter will reach its base: the minimum amount measured with a Sun altitude of 30° made up 0.003 cal. The appearance at the zenith of a ridge of such a cloud may, on the contrary, yield a powerful flux of scattered radiation: in our observations the maximum was 0.119 cal. when measured at a Sun height of 40°.

When examining the table, the eye is struk by the fact that with a Sun altitude of 30°, the value of *B* for *Cu* is equal to 2.4, while it attains 7.8 with an altitude of 40°. For none of the other cloud shapes is such an important difference observed.

This should be accounted for by the fact that with a Sun height of 40° the measurements were carried on at hours approaching midday, when the *Cu*

clouds are at the maximum of their development, while with a Sun height of 30° we observe either the initial phase of their development (morning hours), or the phase of their decline previous to vanishing (evening hours), which tells upon the minima of radiation.

Within some groups of clouds it is possible to discriminate between separate varieties differing as to their radiation values. Thus, within the group *Ci*, the highest values are obtained for the variety *cirrus densus*, and the lowest for *cirrus filosus*.

With clouds of the *Ac* group, the highest values correspond to *altocumulus cumulogenitus*, while the lowest characterize *altocumulus translucidus*. Up to the present time we did not succeed clearly to bring out from the data obtained the effect of the snow-cover albedo upon the quantity of radiation.

The table contains absolute figures for the maxima and minima of radiation; since, however, no sufficient number of observations has still been carried out, these figures may fail to characterize the respective groups of clouds. In order to better characterize the deviations of the maxima and minima from the means, the latter are plotted in the graph as horizontal lines, while deviation to either side is indicated by shading. The maxima and minima have been obtained as the means from five maximum or minimum figures for every cloud group. The figure goes well to illustrate the radiation properties of the clouds investigated.

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TECHNICAL PHYSICS

DIELECTRIC PERMEABILITY OF RUTILE MIXTURES

By B. M. WUL, Corresponding Member of the Academy

Rutile is distinguished from other substances by its high value of dielectric permeability. For polycrystalline rutile ε equals 114 (1). A. F. Joffe suggested utilization of rutile mixtures in order to avoid the edge discharge at high voltage. In fact, measurements performed by the author in collaboration with A. F. Joffe (2) and later with I. M. Goldman (3) have shown that the application of mixtures containing rutile, as well as the protection of electrode edges with the aid of films prepared of rutile mixtures, lead to a considerable increase of breakdown voltage or the flash-over voltage, sometimes as much as 2—3-fold.

By the way it was shown that the dielectric permeability of rutile mixture with linseed oil is connected with the concentration of rutile in the mixture

$$\log \varepsilon = \vartheta \log \varepsilon_1 + (1-\vartheta) \log \varepsilon_2 \qquad (1)$$

where ε, ε_1 and ε_2 are the dielectric permeabilities of the mixture, rutile, and linseed oil, respectively; ϑ is the proportion of rutile in the mixture. This relation stated by Lichtenecker (4) was later proved for rutile mixtures in other investigations as well (5—6).

Recent years have witnessed the beginning of the application of rutile as a constituent part of the material of which high frequency ceramic capacitors are made. It is only natural that Lichtenecker's relation stated for ordinary mixtures breaks down in case rutile reacts chemically with the other components of the mixture. Investigation of compounds of TiO_2 with MgO, performed by the author in collaboration with Barzakovsky, Dubrovo and Skanavi, has shown that when the concentration of TiO_2 in the compound TiO_2—MgO increases up to 45 per cent, the dielectric permeability of the compound changes but little and remains comparatively small, not higher than 13. On the other hand, G. I. Skanavi has found that by using rutile dolomite compounds, substances with as high a dielectric permeability as about 70 can be obtained. Another difference between the TiO_2 compounds with dolomite and magnesium titanates is that the former show a negative temperature coefficient of dielectric permeability, while the temperature coefficient of the latter is positive. If we suppose that the reacting ability of TiO_2 is exhausted in compositions with MgO and dolomite, then we may hope that for the combination of these compounds Lichtenecker's relation (1) holds again.

It follows from it that when temperature t varies, the variation of the mixture's dielectric permeability is connected with the variation of the dielectric permeabilities of the components by the relation

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} = \vartheta \frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dt} + (1-\vartheta)\frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{dt} \qquad (2)$$

The quantity $\frac{1}{\varepsilon}\frac{d\varepsilon}{dt}$ is called temperature coefficient of dielectric permeability.

Thus, in order to determine the temperature coefficient of dielectric permeability for a mixture we can use the usual Newton law for the properties of a mixture. The temperature coefficient of dielectric permeability of a mixture equals to the weighed arithmetical mean of the temperature coefficients of dielectric permeability of the components. Measurements performed by G. I. Skanavi, which are given in Figs. 1 and 2, support this regularity.

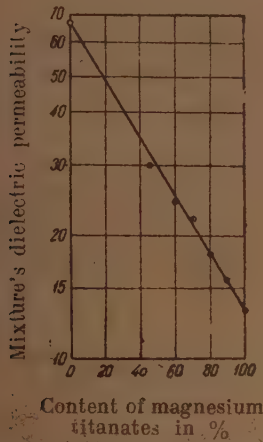

Fig. 1. Dielectric permeability of titanate mixtures

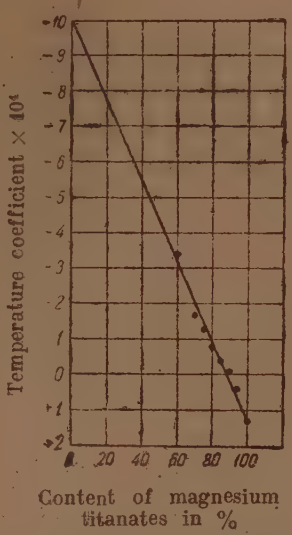

Fig. 2. Temperature coefficient of the dielectric permeability of titanate mixtures

The possibility of controlling the temperature coefficient of capacity is very important in practice, especially in radiotechnical applications, for obtaining oscillatory circuits whose frequency is invariable with temperature.

From equations (1) and (2) follows:

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} = A\log\varepsilon - B$$

where

$$A = \frac{\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dt} - \frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{dt}}{\log\varepsilon_1 - \log\varepsilon_2},$$

$$B = \frac{\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dt}\log\varepsilon_2 - \frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{dt}\log\varepsilon_1}{\log\varepsilon_1 - \log\varepsilon_2}.$$

The subscripts 1 and 2 refer to the corresponding values of the mixture components.

For $B = A\log\varepsilon$ the temperature coefficient of dielectric permeability vanishes. The corresponding value of dielectric permeability of the mixture is determined by the equation:

$$\log\varepsilon = \frac{\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dt}\log\varepsilon_2 - \frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{dt}\log\varepsilon_1}{\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dt} - \frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{dt}}$$

The investigation of the dielectric properties of rutile compounds was performed in the Laboratory of dielectrics of this Institute, at first in collaboration with the Silicate group of I. N. Grebenshchikov from the Institute of General and Inorganic Chemistry, then on one of the plants of People's Commissariat of Electrical Industries (director of the plant I. M. Kalashnikov).

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BIOPHYSICS

CONTRIBUTION TO THE THEORY OF IMAGE FORMATION ON THE RETINA

By D. MAKSUTOV

(Communicated by S. I. Wawilow, Member of the Academy, 8.I. 1941)

The human eye is not a perfect optical system. The mosaic texture of the retina, the spherical and chromatic aberrations are known to impair the resolving power of the eye, though to a slight degree. The principle cause of the reduction of the resolving power is, however, the optical heterogeneity of the lens; the latter was given no due attention by writers on the subject, except for Helmholtz.

The irregular and unstable shape of the lacrymal film over the cornea, dust particles coming from the air, which produce its local deformations, as well as the vibration of the eye globe and of the sight line, result in additional loss in the resolving power.

The present note is dealing with the principal agent, the heterogeneity of the lens; moreover, it contains an account of the simplest methods for observing the heterogeneity of the lens and for measuring the effective aperture.

All the observations refer to the writer's right eye and should be repeated on a large number of observers, in order to find out the eye properties common to the average observer.

§ 1. Heterogeneity of the Lens. A distant luminous point is perceived by an eye accomodated to it as bright nucleus of irregular shape, surrounded by a halo (Fig. 1, *a*). The stretched and asymmetrical shape of the image is

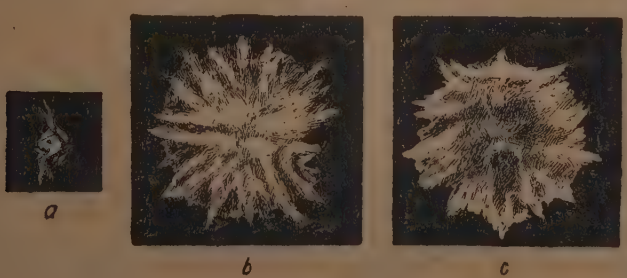

Fig. 1

accounted for by irregular astigmatism of the eye; the astigmatism can be eliminated with the aid of an appropriate cylindrical spectacle glass. Irregularities of the astigmatism will remain uncorrected.

Let us now observe the same point with the aid of a lens of +2 dioptries, together with another serving to compensate for the eye's astigmatism. In this case the image is formed within the eye anteriorly to the retina upon which a spot is projected (the section of the light cone behind the focal point) (Fig. 1, *b*). Let us make an accurate drawing of the out-of-focus image, measuring at the same time the effective aperture of the eye (d_r) by the method to be described below.

In the same way let us arm the eye with a lens of about—2 dioptries, and draw the out-of-focus spot (section of light-cone before focus) (Fig. 1, *c*).

If the eye were a perfect optical system, pictures *b* and *c* would appear as discs of uniform brightness. As a matter of fact, it is not, and the wave front refracted by the media of the eye, inclusive of the lens, ceases to be spherical, but suffers general and local deformations instead. Every crest of the wave front is visualized in picture *b* by a local light condensation, while in *c* it is represented by a light deficiency, and *vice versa* in the case of depressions of the wave front.

Picture *b* differs from *c* in that, first: in the former the centre is brighter than the periphery, while in the latter it is the periphery which is brighter; this is evidence of a n e g a t i v e s p h e r i c a l a b e r r a t i o n being present in the eye.

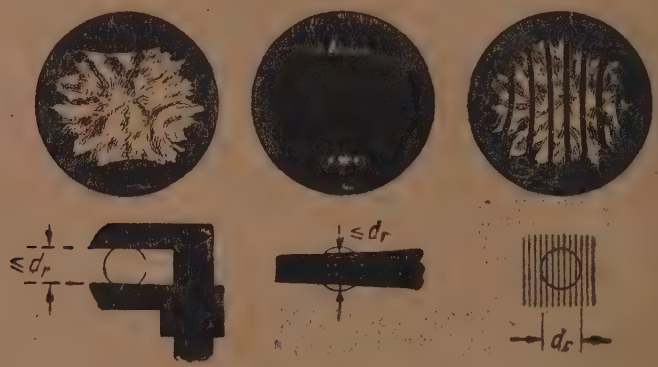

Fig. 2

Secondly, in picture *b* the periphery shows a bluish colour, while in *c* it is reddish, which points to a presence of an u n c o r r e c t e d c h r o m a t i c a b e r r a t i o n.

But, which is most typical of the pictures *b* and *c* is the diversity and conspicuousness of the local condensations and deficiencies of light, which are evidence of considerable l o c a l o p t i c a l h e t e r o g e n e i t y of the eye media and in the first line, of the eye-lens.

§ 2. M e a s u r i n g t h e E f f e c t i v e A p e r t u r e o f t h e E y e. The out-of-focus pictures of Fig. 1 should be supplied with a scale. For this purpose the effective aperture of the eye is to be measured.

In what follows three methods for doing so are being described.

1) The eye is armed with a lens of about +2 dioptries and views the picture *b* of Fig. 1. A caliper is brought in direct proximity to the eye, its arms being gradually drawn together, until the first truncation of the rays of the fringe appears in picture *b* at two diametrically opposed points. A reading on the caliper gives the value of d_r.

2) Under the same observational conditions, an opaque wedge or a cone with acute apical angle is placed in front of the eye instead of the calipers. The first appearance of luminous radiant fringes will correspond to the equality of d_r and the width of the wedge obstructing the pupil. This method is negative with respect to the first one.

3) Under the same observational conditions a grating is placed in front of the eye, and shadows of the lines are observed on picture *b*; d_T is estimated after the number of intervals between the lines of the grating.

All the three procedures are explained in Fig. 2.

By applying any of the procedures here described, d_T can be measured in the dark under any adaptation desired. The first two, however, require for taking readings either turning-on light, or the help of an assistant, whereas the third method enables the observer himself to take readings, in the dark at that.

Besides, the third method makes it possible to bring out the distortion of the scale in the out-of-focus pictures, due to the spherical aberration of the eye, and thus to measure, be it even roughly, the magnitude of the eye's spherical aberration.

A scale is thus supplied to picture 3 of Fig. 2; it is on this scale that the extension of heterogeneities is estimated. Thus, in the central region of the crystalline lens the local heterogeneities are of the order of 1 mm (distances between the crests of the wave surface deformed), while on the periphery of the lens the heterogeneities are of the order of 0.6—0.7 mm, being coarser, besides.

It is perhaps this fact which accounts, partly at least, for the effect of Stiles and Kraufords.

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PHYSICAL CHEMISTRY

ANOMALOUS CASES OF ELECTRODE POTENTIALS OF SOLID SOLUTIONS

By G. V. AKIMOV, Corresponding Member of the Academy, and G. B. CLARK

The electrode potentials of solid solutions usually hold intermediate position between the potentials of the components. Further we shall conventionally describe the component more negative electrically as anodic, and the more positive component, as cathodic.

In the constitution—potential diagram the value of the potential of a solid solution may for different reasons which are not altogether clear as yet, be shifted from the right line connecting the potential values.

Among the causes responsible for this one may point out the influence of protective films, the polarization, the elimination of atoms of the anodic component under the action of the solution, a secondary deposition of the cathodic component, the energy of formation of the solid solution.

Heretofore, however, the existence of anomalous cases of solid solution potentials was not even surmised. We mean the cases where the addition of the anodic component instead of lowering the potential increases it, or where the addition of the cathodic component decreases the potential of the solid solution, though it should normally be expected to increase it.

These two kinds of potential anomaly were disclosed by us while investigating systematically the potential of solid solutions on the basis of aluminium, magnesium, zinc, copper and silver in typical electrolytes.

To the first anomalous type belong the systems: Mg—Tl, (Mg—Tl), (Mg—Cd), (Mg—Zn) in a 3 per cent solution of NaCl; Mg—Tl, (Mg—Tl), Mg—Al, (Mg—Al) in a 0.1 *n* solution of HCl; Mg—Al, (Mg—Al) in a 0.1 *n* solution of HNO_3; (Mg—Zn) in a 0.1 *n* solution of NaOH. Enclosed in parentheses are the systems measured with the aid of the technique we have designed for measuring the electrode potential under continuous abrasion of the metal surface while under solution (1).

Fig. 1 is a diagram of Mg—Tl electrode potentials measured with and without abrasion. For the components of this system our measurements gave the following values of electrode potential:

Without abrasion	E_{Mg}	—1400 mV
» »	E_{Tl}	— 440 mV
With abrasion	(E_{Mg})	—1520 mV
» »	(E_{Tl})	— 500 mV

It will be apparent that whether measured with or without abrasion, the potential of Tl in the case of this system is nearly 1 Volt more positive than that of Mg.

In spite of so great a difference in the potentials, the result of the introduction of atoms of the more positive Tl into the lattice of the electrically negative Mg is that the potential of Mg sharply decreases in nobility. If about 1.5 per cent of Tl

by volume are introduced into the solid solution, the potential of the alloy proves to be nearly 300—400 mV more negative than the potential of Mg.

In Fig. 1 the straight lines *a* and *a'* connect the potentials of pure magnesium and pure thallium.

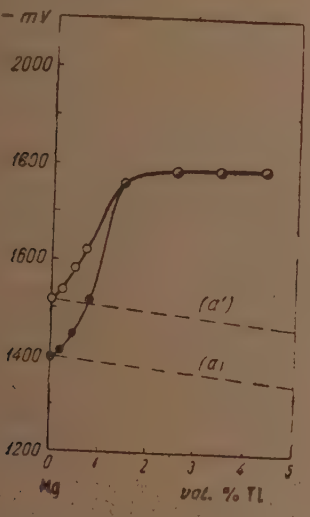

Fig. 1. Electrode potential of solid solution of Mg — Tl in 3 per cent solution of NaCl

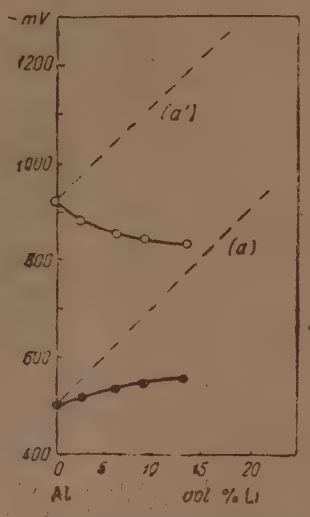

Fig. 2. Electrode potential of Al—Li solid solutions in 0.1 *n* HCl

In the second anomalous type belong the systems (Al—Mg) in a 3 per cent solution of NaCl; (Al—Li), (Al—Mg), (Cu—Sn) in a 0.1 *n* solution of HCl; (Al—Li), (Al—Mg) in a 0.1 *n* solution of HNO_3; (Al—Mg), (Cu—Al), (Cu—Mn) in a NaOH solution. A typical example is the system (Al—Li) represented in Fig. 2. The electrode potentials of its components were found to be as follows:

Without abrasion	E_{Al}	— 500 mV
» »	E_{Li}	—3000 mV
With abrasion	(E_{Al})	— 920 mV
» »	(E_{Li})	—3000 mV

Thus, the introduction of the very negative lithium into the aluminium lattice leads to an increase of the potential of the alloy. How much the real curve deviates from the ideal course (right line *a'*) is seen on the diagram. It may be remarked that although for the same system without abrasion (Fig. 2) the potential does not increase and even becomes somewhat less noble, yet the comparison of the real run of the curve with the straight line *a* shows that the difference accounted for by abrasion is not very material.

If one now turns to the causes responsible for these anomalous cases, it should be taken into consideration that both the polarization phenomena and the dissolution of the anodic component and the secondary deposition of the cathodic component cannot possibly alter the potential of the solid solution beyond the range of values of the components' potentials. So strong a change of potential may possibly be connected with the sharp alteration of properties of the protective film. In that case the potential of the solid solution can surpass the limits of the potentials of the components, but, of course, only if the value of the potential of at least one of the components is greatly increased, owing to the presence of the protective film. Without giving up this explanation in certain concrete cases (for instance, for the system Mg—Tl), we cannot consider it to be fundamental, since a considerable number of anomalous systems are just systems with abrasion in which the rôle of the protective film is reduced to a minimum.

Another assumption explains the anomaly by a strong anodic polarization of the surface produced by some cathodic impurities of the alloy (for instance, Fe in Al) which are not taken into account. Owing to this, the measured potential of the alloy is shifted towards the positive side. If the addition of the cathodic component will lead, because of the formation of compounds with impurities, to a sharp increase of overvoltage on the impurities, then the potential of the alloy will be shifted into the negative region. Thus, addition of the cathodic component makes the solid solution less noble (and at the same time the rate of corrosion is decreased). It does not seem likely, however, that the scale of the observed alterations of the potential may correspond to the operation of this factor.

Finally, one may suppose the reason to lie in a sharp change (conversion) of the electro-chemical properties of the metals upon the formation of the solid solution consequent upon the sharp alteration taking place in the electronic properties of the alloy. It is thus probable that lithium, while in the solid solution with aluminium, behaves virtually as a cathodic, rather than anodic component.

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PHYSICAL CHEMISTRY

ON THE ROLE OF CATALYSIS IN THE PROCESS OF SELF-IGNITION

By N. S. AKULOV, Member of the Academy of Sciences of the White Russian SSR

In one of our previous papers (1) we showed that self-ignition sets in upon the expiration of the induction period, determined by the relation

$$\tau = \frac{\Delta\tau_0}{\omega - 1} \qquad (1)$$

where ω is the mean number of new chain links arising directly from each preceding one, and $\Delta\tau_0$ is the time of development of the initial link. Here ω is equal to $\sum_i \varkappa_i \delta_i$, where δ_i is the probability of the formation of a link followed by $\varkappa_i$ new links.

Expression (1) may be very simply derived on the basis of the following analysis of mechanism of the development of chain processes for which $\omega \geqslant 1$*. For the sake of convenience, we may by rearranging the links (not changing their number on the average) represent any such branched chain having rupture links (Fig. 1, *a*) in the form of segments of continuously developing simple chains, which we will call branches of the continuous avalanche (Fig. 1, *b*). All that is necessary that the avalanche have the same value of ω as the initial chain.

Fig. 1. a) $\omega = \Sigma \varkappa_i \delta_i = 0 \cdot \frac{3}{12} + 1 \cdot \frac{2}{12} + 2 \cdot \frac{5}{12} + 3 \cdot \frac{2}{12} = \frac{3}{2}$; b) $\omega = \frac{3}{2}$; $\mu = 2$

Inasmuch as the induction period is the time that elapses from the start of the reaction to the moment its rate begins to increase, it is equal to the mean time of development of the first branch of the avalanche. If this branch consists of μ links

$$\tau = \mu\Delta\tau_0 \qquad (2)$$

But according to the definition of ω we have for an avalanche

$$\omega = \frac{(\mu - 1) \cdot 1 + 1 \cdot 2}{\mu} \qquad (3)$$

* For simplicity, we consider here the case $\omega \leqslant 2$. Generalization for $\omega > 2$ does not present any difficulties whatsoever, but is not necessary for what follows.

where

$$\mu = \frac{1}{\omega - 1} \tag{4}$$

From (2) and (4) we indeed get (1)*.

From (1) we see that self-ignition is possible only in the region $\omega \geqslant 1$. At the boundary of this region the induction period is infinite and the branched chain becomes a simple chain of infinite length (we note that as a result of the fluctuation of ω about the value $\omega = 1$, the rate of the process here will be practically equal to zero).

In the first approximation the reaction rate is determined, as we showed earlier, by the relation

$$W = \frac{c(\alpha)}{\Delta\tau_0}(\omega - 1) \tag{5}$$

where $c(\alpha)$ is a coefficient dependent upon the stage of the reaction ($0 \leqslant \alpha \leqslant 1$). For any stage, in particular, the stationary stage (5) gives $W_{\omega \to 1} \to 0$. To calculate ω in a more general case we must take into account that a chain may consist of links of a diverse type (ij). Let δ_{ij} be the probability of the formation in a chain of a link of the ij-th type, and $\varkappa_{ij}$, the number of new links that it produces after its formation. Then

$$\omega = \sum_{i,j} \varkappa_{ij} \delta_{ij} \tag{6}$$

The links for which $\varkappa = 0$, $0 < \varkappa < 1$ and $\varkappa > 1$ will be called rupture links, development links or branching links, respectively, and the quantity $\varkappa$, the branching factor.

Links of the ij-th type can develop only during effective collisions (*i. e.*, collisions leading to reaction) of the molecules of the i-th intermediate product with molecules of the j-th initial product or with the surface of the catalyst. Let Z_{ij} be the number of a definite type of effective collisions, and $Z = \sum_{ij} Z_{ij}$, the total number of effective collisions for the same time interval. Then

$$\delta_{ij} = \frac{Z_{ij}}{\sum_{i,j} Z_{ij}} \tag{7}$$

Let us denote the molecules of the initial product by A_j, the molecules of the intermediate product or active centres by X_i, and the molecules of the final product by B_i. Let M stand for any molecule.

Let us, first of all, consider the influence of a heterogeneous catalyst on the course of the reaction. Its surface S consists of a number of areas S_j, on which different types of molecules of the initial, final or intermediate product are adsorbed. The number of effective collisions Z_{ij} with each such area is determined by the relation

$$Z'_{ij} = a'_{ij} [X_i] S_j e^{-E'_{ij}/RT} \tag{8}$$

where E'_{ij} is the energy of activation; $[X_i]$ is a coefficient of the given type X_i; a'_{ij} is the coefficient proportional to the steric factor and which may depend in addition upon the coefficient of diffusion and upon the concentration c of the substance, if in the proximity of the catalyst there is a layer with a changing concentration (the case where we have to do with a gradient

* From (2) and (12) [1] we find under these conditions that $\nu^* = \mu$.

concentration c).

Further we will have links developing as a result of double collisions of active centres with the initial product. Their number is given by

$$Z''_{ij} = a''_{ij}[X_i][A_j]Ve^{-E''_{ij}/RT} \tag{9}$$

where V is the volume of the reaction space. Finally, for links forming as a result of triple collisions we have

$$Z'''_{ij} = a'''_{ij}[X_i][A_j][M]Ve^{-E'''_{ij}/RT} \tag{10}$$

where $[M]$ is the concentration of the «third partner».

These numbers of diverse chain links are easily expressed by the total (initial) concentration of the substance c and the coefficients γ_j characterizing the relative initial concentrations of the initial products $[A_j]/c$. For a two-component mixture we have in particular:

$$\gamma = [A_1]/c \text{ and } 1-\gamma = [A_2]/c \tag{11}$$

As regards S_j, the magnitude of these areas proportional to S and dependent upon c may be determined with the aid of Langmuir's isotherms. In the case of saturation adsorption their magnitude will, generally speaking, not depend upon the concentration c, but, nevertheless, the magnitude of S_j may depend upon the composition of the mixture, being a linear (or more complex) function of γ.

The concentrations $[X_i]$ $[X_j]$ will, generally speaking, be unknown functions of c, α and γ. In what follows, however, we will consider so-called «synchronous» chain reactions, for which in accordance with their definition

$$\frac{[X_i]}{[X_j]} = \text{const} \tag{12}$$

i. e. the ratios of the concentrations $[X_i]$ and $[X_j]$ do not depend either upon c, or α, or γ. When the relations (12) and (8)—(10) are the case, the dependence of δ_{ij} upon c and γ is found to be particularly simple*.

On the basis of (6) and the subsequent relations for the case when we have only a negative catalyst, *i. e.* $\varkappa'_{ij} = 0$ and $\varkappa_{ij}$ is different from zero only for double collisions ($\varkappa''_{ij} \neq 0$, but $\varkappa'''_{ij} = 0$), we get:

$$\omega = \frac{\sum_{ij} \varkappa''_{ij} Z''_{ij}}{\sum_{ij} Z'_{ij} + \sum_{ij} Z''_{ij} + \sum_{ij} Z'''_{ij}} \tag{13}$$

Under certain conditions the factor $\varkappa''_{ij}$ may turn out, just like δ_{ij}, to be a function of c and γ. This is just what we find when we are dealing with the effect of stimulating the branching links with the rupture links (in the volume)**. And indeed, if the negative catalyst, for instance, the walls of the reaction vessel, is kept at a constant temperature, the energy liberated when X_i collides with it will pass out of the system. At the same time the energy liberated in triple collisions remains within the reacting system and, thus, increasing the possibility of the formation of branching links during double collisions, enhances the reacting ability of the mixture. In accordance with this, denoting the increase in $\varkappa''_{ij}$ by $\Delta\varkappa''_{ij}$, we may assume that $\Delta\varkappa''$ varies in direct proportion to Z''' and in inverse proportion to Z' (in accordance with the condition that

* We note, however, that generalization for the case $[X_i]/[X_j] \neq \text{const}$ (and also for the case of positive catalysis or other types of developing and branching links) on the basis of the method developed here does not present any difficulties.

** For other cases of interaction see [2] and [3].

when $Z''' \to 0$ or $Z' \to \infty$ we have $\Delta x'' \to 0$). Hence we get

$$\Delta x''_{ij} = \frac{\sum_{kl} q'''_{kl} Z'''_{kl}}{\sum_{kl} q'_{kl} Z'_{kl}} \qquad (14)$$

where q_{kl} are coefficients characterizing the «effectiveness» of the collisions in the above sense and thus depending upon the magnitude of the energy that is absorbed or liberated during collisions.

On the basis of (13) the equation of the boundaries of the region of self-ignition $\omega = 1$, with $\Delta x''_{ij}$ taken into account, is written in the following form.

$$\sum_{ij} (x''_{ij} + \Delta x''_{ij}) Z''_{ij} = \sum_{ij} Z'_{ij} + \sum_{ij} Z''_{ij} + \sum_{ij} Z'''_{ij} \qquad (15)$$

Whence, after substituting the corresponding values of Z_{ij} from (8)—(10) and $\Delta x''_{ij}$ from (14) and discarding for simplicity the difference in E''_{ij}, *i. e.* assuming that $E''_{ij} = E_2$ and, correspondingly, that $E'_{ij} = E'''_{ij} = E_1$*, we get, denoting the difference $E_2 - E_1$ by E,

$$(1 + a_3 D c^2)\, e^{-E/RT} = a_1 D^{-1} c^{-1} + a_2 c \qquad (16)$$

where $D = \frac{V}{S}$ and where $a_k (k = 1, 2, 3)$ are rational functions of the type**

$$a_k = \frac{a'_{0k} + a_{1k}\gamma}{b_{0k} + b_{1k}\gamma} \qquad (17)$$

Here, however, if the reactions of desactivation of triple collisions depend upon the form of the «third partner» M, terms of the second power ($\sim\gamma^2$) enter into the numerator of the expression for a_k. Let

$$c_m = \sqrt{\frac{a_1}{a_2 D}}\,;\; c_s = \frac{1}{\sqrt{a_3 D}} \text{ and } \frac{E}{RT_m} = \ln \frac{c_s^2 + c_m^2}{2\sqrt{a_1 a_2 c_s^2}} \sqrt{D} \qquad (18)$$

Then (16) may be rewritten as follows:

$$2\left(1 + \frac{c^2}{c_s^2}\right) f = \frac{c}{c_m} + \frac{c_m}{c} \qquad (19)$$

where

$$f = \frac{c_s^2}{c_s^2 + c_m^2}\, e^{E/R(1/T_m - 1/T)} \qquad (19')$$

As regards the physical meaning of c_m and T_m and also of c_s and T_s, they are, as we showed earlier, the coordinates of the points determining the beginning and end of the second limit in a c, T diagram. Here T_s is determined from (10) after substituting $c = c_s$.

The equations (18) and (19) lead to the following conclusions.

With a change in the ratio of the surface S of a negative catalyst to the volume of the reaction space:

1) The scale of the boundary curve for the region of self-ignition along the axis of concentrations $c \sim \frac{p}{T}$ increases proportional to $D^{-1/2} = \sqrt{\frac{S}{V}}$ (in particular, this scale for the case of negative catalysis, due to the walls of the reaction vessel, is inversely proportional to the square root of the diameter of the vessel).

2) The boundary curves are shifted along the temperature axis according to the relations:

$$E/RT_m = a_m(\gamma) + n \ln D \text{ and } E/RT_m = a_s(\gamma) + n \ln D$$

where $n \approx 1/2$.

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* The possibility of such a simplification must be tested by experiment. Otherwise it is not difficult to write relations taking into account different values of E'_{ij} and E''_{ij}, E'''_{ij}.

** We confine ourselves here to two-component systems.

Experimental data available in the literature [for example, for CO and O_2(4) and for H_2S and O_2(5)] confirm these theoretical conclusions. The shift along the temperature axis actually satisfies the given relations, where n, in order of magnitude, is close to 1/2.

The change in scale along the p/T axis can be conveniently checked quantitatively against $\frac{p_s}{T_s}$ values. From (18) when $\gamma = \text{const}$ it follows that $\frac{p_s}{T_s}\sqrt{D} = \text{const}$. As the table computed according to Yakovlev's data shows, this theoretical conclusion checks quantitatively with experiment.

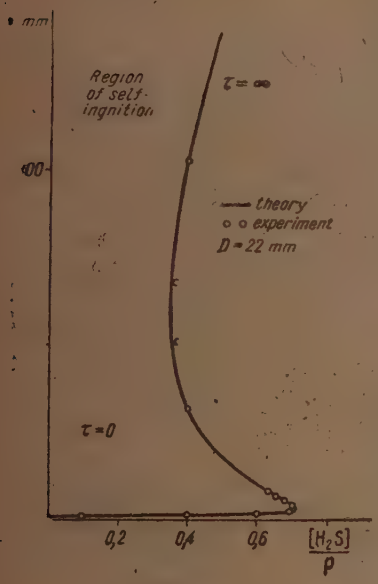

Fig. 2

D', cm	T_s, °K	p_s, mm	$\frac{p_s}{T_s}\sqrt{D'}$
0.92	705	110	0.14
2.2	655	66	0.15
4.0	620	47	0.15

To complete this analysis of self-ignition limits we now have only to compute the shape of the boundaries on the diagram p, γ at $T = \text{const}$, $D = \text{const}$. This may be done with the aid of two equations (of the second order) $p_i = f_i(\gamma)$, obtained from (19) when: 1) $\frac{c^2}{c_s^2} \to 0$ and 2) $\frac{c_m}{c} \to 0$. As empirical constants it is sufficient here to introduce for the first equation the coordinates of a point (p_m, γ_m) and for the second, the coordinates of a point (p_s, γ_s)*. The theoretical boundary curve of the region of self-ignition for the values $a_1 \simeq \gamma^{-1}$, $a_2 \simeq (1-\gamma)\gamma^{-1}$ and $a_3 \simeq 1-\gamma$ that is obtained is given in Fig. 2. As we see, a quantitative agreement with experiment is attained.

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* The points p_m and p_s correspond to the double roots of the given equations of the second order, which have the following form:

$$p = 8.7\,\frac{\gamma}{1-\gamma}\left(1 \pm \sqrt{1-0.13\,\frac{1-\gamma}{\gamma^2}}\right) \text{ and } p = \frac{36}{\gamma}\left(1 \pm \sqrt{1-0.83\,\frac{\gamma^2}{1-\gamma}}\right).$$

The «joint» point of these two curves as chosen by us corresponds to $p = 12-15$ mm [this is the point of inflexion of the curve (19)].

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GEOCHEMISTRY

ON THE CONDITIONS UNDER WHICH NATURAL WATERS BECOME ENRICHED WITH RADIUM AND ITS ISOTOPES

By E. S. STCHEPOTJEVA

(Communicated by V. G. Chlopin, Member of the Academy, 8.X.1943)

Natural waters enriched with radium, *i. e.* such as contain 10^{-11} g of Ra per liter and more, are known by the name of radium-bearing waters.

The possibility in some cases to utilize these waters on industrial scale awake a lively interest to them, which resulted in numerous investigations. Owing to these, some general principles concerning radium-bearing waters could be established (1).

As a next step in the investigation of radium-bearing waters appeared the experimental works by I. E. Starik (2) and his co-workers. These studies revealed the peculiar mode of occurrence of the radium isotopes in rocks, and gave a general picture of their leaching from the rocks.

The third group of studies devoted to this question consists of both analytical and experimental works carried out by the present author within the last years. Together with the bulk of material obtained by earlier writers, these studies permit to form the following idea as to the enrichment of natural waters with radium and its isotopes.

Rocks of any kind are pervaded by a network of internal «capillaries»* filled with water.

Before the rock comes into contact with the gravitational water, some of these capillaries disposed near the accumulations of radioelements, contain radium isotopes, partly in solution, partly in adsorbed condition at the walls of the capillaries; others practically contain none. As soon as the rock comes into contact with the gravitational water which previously contained no Ra isotopes, these elements begin to diffuse into the gravitational water, and from the latter into the water of those capillaries that did not contain them before. The concentration of Ra and its isotopes in the gravitational water depends on the value and mode of variation of those two flows in time and in space. If the gravitational water is not moving, an equalization of non-uniformity, which is rather common in nature, takes place: the capillaries filled with Ra become poor in it, while those which failed to contain it before, become enriched. But if gravitational water is in movement, then at the initial part of the route, where the concentration of Ra and of its isotopes is still insignificant, an intense extraction of the radioelements from the capillaries they fill up, and a weak filling up of the empty capillaries takes place.

The rock becomes in this way poor in radioelements, while deep-lying atom, or such as are contained in very narrow capillaries begin gradually to diffuse into the gravitational water. And the longer a given portion of a rock will be in con-

* Disturbances in the crystalline lattice, fissures, vacuities, which may possibly not look like capillaries.

tact with the water moving by, the wider* is the zone of the rock becoming poor in radioelements, and the more time will they take to migrate from the depth of the rock into the gravitational water. In this case long-lived Ra isotopes will, of course, find themselves in a more advantageous position with respect to their transition into water, than short-lived ones, *viz.* $MsTh_1$, AcX, ThX. In the medium part of the path, according as the water advances and the concentration of radioelements in it increases, the capillaries that have previously been filled up become poor at a lower rate, and those which were poor in Ra isotopes become enriched more intensely: the zone within which the rock looses the radioelements becomes restricted, while that within which it is enriched widens. Lastly, such a condition is also conceivable, in case the water path is sufficiently long, where the previously empty capillaries will prove overfilled (at a given concentration of the solution). The quantity of the radioelement supplied from the gravitational water will then be equal to that which undergoes decay in the depth. This is the least value conceivable of the flow of the radioelement inside the rock. The supply of radioelements from the depth of the capillaries that have previously been filled with radioelement will practically come to an end because of the high concentration of the radioelements in the gravitational water, *i. e.* these capillaries will remain filled in spite of the presence of gravitational water. As to the part of atoms of the radioelements, which are contained in the capillaries in the closest proximity to the gravitational water and are capable in the first approximation to partake in the adsorption exchange (ν_*), they will partake in it independently of and along with those atoms which form the adsorption layer throughout the channel, along which the water is moving. In this case the quantity of the radioelement supplied to the water will be at its maximum, but the rock will not become poor in it, for, adsorption equilibrium being attained, the amount of radioelement which will settle from the water upon the rock will be equal to that given off by the latter.

The formation of radium-bearing water thus portrayed may be represented in the shape of mathematical equations; these equations should of course be considered only as the first attempt to formulate a theory of formation of radium-bearing waters, giving but a qualitative characteristic of the values and the trend of the process responsible for the content of Ra and its isotopes in these waters.

Let us imagine schematically gravitational water to move along a certain channel through the rock. Let n be the number of atoms of the radioelement in the water filling a certain section of this capillary; ν, the amount of the radioelement adsorbed on the surface of this section; ν_1, the amount of the radioelement that diffused into the depth of the rock through this surface; M, number of ions settling by adsorption from the water on this surface per second; k, the coefficient of desadsorption ($\frac{1}{k} = \tau$ is the mean lifetime of an ion in adsorbed state; $k\nu$, the number of ions washed-off from the surface into water per unit time); λ, the constant of decay of the radioelement; β, the coefficient characterizing the diffusion into the depth of the rock (on the assumption that the number of atoms diffusing per unit time from the surface of the rock into its depth is equal to $\beta\nu$); m, number of atoms supplied per unit time to the gravitational water from the rock. In the initial and middle part of the path of the water, m is the number of atoms supplied to it from the depth of the capillaries, obeying therefore the laws of diffusion. In the final part of this path, provided it is sufficiently long, m is the number of atoms supplied to the water from the parts of the capillaries nearest to the surface; it obeys therefore the laws of adsorption: $m = k\nu_*$; $t = \frac{s}{\varphi}$ is the time of contact of each element of the water with the rock (s is the length of the path; φ, rate of filtration); θ is the time of contact between the rock and the water that is continuously moving by, the lifetime of a given water path; $\gamma = \nu/n$ is equal to the ratio of concentrations $\frac{c_{ads}}{c_{sol}}$ multiplied by the ratio of rock surface to water volume contained in it.

* Dimensions of the order of small divisions of a millimeter are understood.

For a cylindrical channel with a radius r $\gamma = \frac{2}{r}\frac{c_{ads}}{c_{sol}}$; γ characterizes the degree of adsorption of the radium and the macroscopic structure of the rock at the same time; $\delta = \frac{\nu_1}{\nu}$ characterizes the internal structure of the rock, *i. e.* the development of the network of the internal capillaries pervading it.

Taking account of what has been said above, we can then write:

$$\frac{d\nu}{d\vartheta} = M - k\nu - \lambda\nu - \beta\nu;$$

$$\frac{dn}{d\vartheta} = m + k\nu - M - \lambda n - \frac{dn}{dt};$$

$$\frac{d\nu_1}{d\vartheta} = \beta\nu - \lambda\nu_1.$$

From the first two equations we obtain by way of some simplifications experimentally checked:

$$\frac{dn}{dt} = m - (\lambda + \beta)\gamma n$$

In its general form this equation now defies solution. For the limiting case, however, when $\frac{dn}{dt} = 0$, we have:

$$n = \frac{m}{(\lambda+\beta)\gamma} \qquad (1)$$

If in this case $\frac{d\nu_1}{d\vartheta} = 0$ too, which may take place with great ϑ, then

$$n = \frac{m}{\gamma\lambda(\delta+1)} = \frac{k\nu_*}{\gamma\lambda(\delta+1)} \qquad (2)$$

The quantities β, γ, δ, k and ν_* can be determined under laboratory conditions for different rocks and waters of various chemical composition.

From the formulae given above, as well as from the whole of the picture of the formation of radium-bearing water, we are able to draw the following conclusions:

1) The quantity n, and consequently the concentration of Ra and its isotopes in water, will be the greater, all other conditions being equal, the higher is their content in the rocks (m, ν_*) and the weaker their adsorption on them (k, γ).

2) This concentration is dependent on the internal structure of the rock (β, δ).

3) The concentration of radium isotopes in water may either depend or not on λ. In case there is a wide zone poor in radium isotopes around a channel with gravitational water, n will depend on λ [see equation (1), where $m = f(\lambda)$ and $\beta \gg \lambda$ and is equal for the various isotopes]. As a result, the ratios $\frac{MsTh_I}{Ra}$ in the waters will be small as compared to the respective magnitudes obtained for the rocks. If there is no wide zone poor in radium isotopes near the issue of the water (great s and ϑ), then the concentrations of Ra and its isotopes in the water will not depend on λ, but only upon their content in the rock, since ν_* [see equation (2)] cannot depend on λ, while δ with ϑ sufficiently great will be inversely proportional to λ because of the dependence of ν_1 on λ.

The relations $\frac{MsTh_I}{Ra}$ in the waters will in this case approach the corresponding quantities obtained for the rocks.

4) Especially advantageous for the appearance of waters with high concentrations of radium and its isotopes are long ways of the water through a rock subject to leaching, but not unless the ϑ are large.

5) Large θ are favourable for the appearance of waters with high concentrations of radium and its isotopes in the case if the path of the water along the rock in process of leaching is sufficiently long. With short paths large θ are not advantageous.

Some of these conclusions have already been supported by analytical determinations of natural waters, others still stand in need for a check-up. But even now, a comparison of the quantities determined under natural conditions $\left(\frac{MsTh_I}{Ra}\right.$ in waters and rocks, $\frac{c_{water}}{c_{rock}}$, $\left.\frac{ThX}{MsTh_I}\right.$ in waters, *etc.*$\left.\right)$ with quantities of k, β, γ, δ and ν_* determined experimentally and under laboratory conditions for a large number of rocks and waters permits of some conclusions as to the conditions of formation of radium-bearing waters, as well as to the probability of existence under given conditions of waters with higher concentrations of Ra and its isotopes.

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BIOCHEMISTRY

ON THE NATURE OF GLOBULAR PROTEINS. I

By S. E. BRESLER and D. L. TALMUD, Corresponding Member of the Academy

The conceptions on the structure of macromolecules which have developed during the last twenty five years are successfully applied to all natural and synthetic high-molecular substances. The only exception is formed thus far by globular proteins, the chief properties of which discovered mainly by Svedberg [1] are as follows: a) in aqueous solutions and hydrated crystals the protein particles have a nearly spherical shape, in spite of the fact of a straight and unbranched polypeptide chain; b) globular protein particles are mono- or paucidisperse, *i. e.* consist of separate molecules with a definite molecular weight or with a few definite molecular weights, which are always integral multiples of a certain minimum value (about 17600); c) on diluting solutions of globular proteins, with a variation of pH of their solutions, on addition of electrolytes, aminoacids or other substances, the globular proteins are reversibly dissociated (and associated), their dissociation products being never smaller than the smallest globular molecule.

The reaction of the formation of globular molecules is undoubtedly determined by statistical laws. The apparent monodispersity of globular molecules is explained only by a sharp maximum of the probability of the distribution curve of the molecular weights. To the commonly accepted principles of the structure of macromolecules we add two new considerations which, without contradicting fundamental principles, enable one to explain the specific properties of globular proteins, which distinguish them from all the known macromolecules. These new considerations consist in the following.

1. Half of the main valency chain of a protein molecule consists of peptide bonds, and these are characterized by a low formation energy (2 500—7 500 calories) [2], comparable to that of the van-der-Waals intermolecular forces.

2. The main valency chain can be disrupted under certain conditions.

These considerations must be consistently applied to the structure of the minimum globular molecules only, since it is proved experimentally that larger globular molecules are associates of small molecules connected with one another by very weak bonds or by a very small number of bonds [1].

The polypeptide chain constituting the smallest protein globula consists of a statistically distributed set of aminoacid residues with diverse side chains. A considerable part (from $\frac{1}{4}$ to $\frac{1}{2}$) of all the side chains are purely hydrocarbon chains. The remaining side chains, being also hydrophobic, contain partly hydrophilic polar groups [3]. From the geometrical point of view the polypeptide chain should have an elongated zigzag shape with alternating side chains directed to opposite sides. This is the only shape which is possible for a polypeptide chain built up for aminoacid residues of the same (left) configuration and for which the side chains do not sterically hinder each other [4]. Under the influence of the heat motion and of the molecular forces between the side chains, the polypeptide chain can be bent by a rotation of separate links of the chain about single bonds, with a resulting

spiral shape. Two circumstances prevent the bending of the chain over large angles: a) as has been proved in the case of hydrocarbon chains, the rotation about simple bonds in a chain is not quite free and is connected with the change of energy (5); b) the configuration of the main valency chain must under any conditions be compatible with the large size of the side chains.

The formation of the globula is realized by the drawing together of hydrophobic side chains which constitute the «nucleus» of the globula. The «envelope» of the globula consists of hydrophilic peptide bonds in the polypeptide chain, curled in a spiral about the «nucleus» and forming its surface.

The total energy of van-der-Waals' bonds between the side chains in the «nucleus» of the globula, which are responsible for the curling of the polypeptide chain, reaches an enormous magnitude. It can be estimated with the help of Langmuir's(6) method of calculating the forces near the surface of the molecule.

This distribution of the side chains between the hydrophobic «nucleus» and the water surrounding the globula must be considered as a statistical process, as a certain dynamic equilibrium depending on the heat motion of the particle, something like the distribution of dissolved molecules between the aqueous and the hydrocarbon phases in contact with each other.

The main valency chain owing to the hydrophilic peptide bonds has a strong cohesion with water.

The volume of the absolutely hydrophobic, *i. e.* purely hydrocarbon side chains can be calculated from the size of the atoms and groups (5).

The mean volume of a hydrocarbon side chain has been determined from the weight of all the hydrocarbon side chains in the total weight of the globula. In the case of haemoglobin this mean value is equal to 160 Å^3 and differs but slightly from other proteins. The cohesive forces in the «nucleus» can be estimated from the average evaporation energy of hydrocarbons, which varies for aliphatic and aromatic hydrocarbons within relatively narrow limits between 80 and 94 cal/g. Referring this energy to 1 gram-mol of the smallest globular particle, we obtain about 500 000 cal. This huge cohesion energy of the absolutely hydrophobic side chains which is hundred times larger than the energy of the formation of the peptide bonds, must play a decisive rôle in the processes of the formation of a globula and in the determination of its final configuration.

The minimum condition for the energy of the fundamental globular molecule consists in the requirement that the hydrophobic «nucleus» should not come in contact with the water, *i. e.* should be wholly covered by the «envelope» of the spirally curled main valency chain. The energy gain due to such a configuration can easily be estimated.

If the «nucleus» of the globula were not protected by the hydrophilic «envelope» and could be treated as a hydrocarbon drop surrounded by water, the drop would possess a free surface energy. Assuming the volume of the «nucleus» of the smallest globula equal to 7000 Å^3, the diameter, 24.4 Å and the area of the surface, 1850 Å^2, we get for a surface tension at the hydrocarbon—water interface 40 erg/cm^2 a total surface energy, referred to a gram-mole of the globulae

$$E = 1850 \cdot 10^{-16} \cdot 40 \cdot 10^{-7} \cdot 0.24 \cdot 6 \cdot 10^{23} = 110\,000 \text{ cal/mol}$$

This explains the fact that the size of the protein globulae is sharply limited, and that the condensation process cuts off practically nearly equal portions of the polypeptide chain. In the globulae of a definite size only there exists a harmonic relationship between the hydrophilic «envelope» formed by the main valency chain and the volume of the «nucleus» consisting of the hydrophobic side chains.

All the polymer homologues of the globular particles will possess a higher energy, since the optimal relation between the volume and the surface of the globula will be destroyed and a surface of contact between the hydrophobic «nucleus» with the water will appear. This is connected with the expenditure of a large specific energy.

The probability of the formation of a protein particle with the given molecular weight will therefore sharply depend on the molecular weight showing a sharp maximum for particles with a minimum total energy.

The disruption energy of the peptide bond enters also as an item in the sum of the formation energy of the globula of a given size, though quantitatively it does not introduce any significant correction. Let us assume, for instance, that the aminoacids form polymers with a degree of polymerization $n+1$ (instead of n). This gives a gain of one peptide bond per n particles; per one globula the average gain of the energy will thus be only u/n, where u denotes the energy of the peptide bond, this of the order of 7500 cal/mol, $n=150$, hence the correction amounts to $\frac{7500}{150} = 50$ cal/mol only. The actual value of the additional surface energy per one side chain immersed in the «nucleus» is $\frac{110000}{50} = 2000$ cal/mol in the case the corresponding portion of the interface with water becomes uncovered.

Let us now estimate the size of the most probable protein globula, and the degree of the statistical spread of molecular weights in the neighbourhood of the equilibrium value. Let us assume that irrespective of the method of formation of protein molecules a statistical equilibrium between globulae of various sizes, according to their energies and the energy of thermal motion, *i. e.* the temperature, is established. Let us assume further that the surface of the globula of the most probable size is equal to $4\pi R^2 = n\sigma$, where n denotes the number of the aminoacid residues or the polymerization degree, while σ_0 is the area of the surface occupied by a single residue, equal to 14 Å^2. The inner volume of the «nucleus $\frac{4}{3}\pi R^3 = \frac{n v_0}{3}$, where v_0 is the average volume of the aminoacid residue equal to 160Å^3.

The ratio of the surface of the peptide bonds and of the volume of the «nucleus» requires

$$\frac{4\pi R^2}{\sigma} = \frac{4/3\,\pi R^3 \cdot 3}{v_0}$$

Hence we get

$$n = \frac{4\pi v_0^2}{\sigma^3} \cong 130$$

The agreement of the calculations with the experiment is very satisfactory (for lactalbumin $n \cong 150$).

The size of the smallest globula is determined by the quantity v_0, the mean volume of the hydrophobic side chain, which depends on the aminoacid composition of the protein. This explains the experimental fluctuations of the molecular weights within the same class for various individual proteins. In the case of a very sharp deviation of the aminoacid composition from the ordinary one, the existence of a minimum globula of a different size is possible.

For the estimation of a statistical spread of molecular weights in the neighbourhood of the most probable one, Bolzmann's function can be used, the probability $W(\Delta n)$ of polymerhomologues with a polymerization degree, differing from the most probable by Δn, being equal to

$$W(\Delta n) = W_0 e^{-\frac{u(\Delta n)}{kT}}$$

where W_0 is the probability of a particle with a polymerization degree n, $u(\Delta n)$ is the additional energy for $n \pm \Delta n$ links. The function $W(\Delta n)$ is normalized in such a way that

$$\int_0^\infty W\,dn = 1$$

The quantity $u(\Delta n)$ can easily be estimated; in first approximation one can put $u(\Delta n) = \Delta n u_0$, where u_0 is the surface energy, corresponding to the introduction of an additional hydrophobic chain in the «nucleus» of the globula.

We have already estimated this quantity which turns out to be equal to $u_0 = \frac{110\,000}{50} \cong 2000$ cal/mol. Hence

$$\frac{W(\Delta n)}{W_0} \cong e^{-\frac{\Delta n\, 2000}{600}} = e^{-10/3\,\Delta n}$$

This is the distribution function of the polymer homologues with respect to the degree of polymerization.

Let us now find the average quadratic deviation from the most probable value of n:

$$\sqrt{\overline{\Delta n^2}} = \sqrt{\frac{\int_0^\infty \Delta n^2 e^{-10/3\,\Delta n}\, d\Delta n}{\int_0^\infty e^{-10/3\,\Delta n}\, d\Delta n}} = 0.3\sqrt{2} \cong 0.4$$

Assuming as before that the «nucleus» contains $^1/_3$ of all the side chains of the globula, we find that the average quadratic deviation of the degree of polymerization is equal to 1.2, *i. e.* but 0.8 per cent of the degree of polymerization of the protein particle. This is the statistical spread of the molecular weight of protein globulae.

Let us now compare the calculated results with the experimental ones.

In the investigations of the sedimentation of the protein in the Svedberg centrifuge the sedimentation boundary appears to be washed off owing to diffusion. The diffusion constant, calculated by Svedberg [1] from the degree of spreading of the boundary, differs by 10—20 per cent from that measured experimentally. This deviation can be due both to the inaccuracy of the method and to the incomplete monodispersity of the protein particles.

We shall estimate the apparent variation of the magnitude of the diffusion coefficient corresponding to the statistical spreading of the molecular weights calculated by us. The spreading of the sedimentation boundary can be calculated in the following way. The equation of the sedimentation of the globulae runs as follows:

$$V = \frac{\omega^2 r v (\delta - 1)}{6\pi\eta a}$$

where V is the sedimentation velocity; ω, the angular velocity; δ, the density of the globula; r, the rotor radius of the ultracentrifuge; η, the viscosity of the solution; a, the radius of the globula; v, the volume of the globula. Substituting $v = {}^4/_3 \pi a^3$ and putting $v = n v_0$ (where n denotes the degree of polymerization and v_0, the average volume of the aminoacid residue), and expressing a and v through the degree of polymerization n, we get

$$V = \frac{\omega^2 r (\delta - 1) n v_0}{6\pi\eta \left(\frac{3nv_0}{4\pi}\right)^{1/3}} = \frac{\omega^2 r (\delta - 1) v_0^{2/3} n^{2/3}}{6\pi \left(\frac{3}{4\pi}\right)^{1/3} \eta}$$

The position of the globula with the polymerization degree n after a certain time interval τ will be

$$x_0 = V\tau = \frac{\omega^2 r (\delta - 1) v_0^{2/3} \tau}{6\pi \left(\frac{3}{4\pi}\right)^{1/3} \eta} n^{2/3} = A n^{2/3}$$

If $n = n_0 + \Delta n$, the coordinate of the particle is

$$x(\Delta n) = An_0^{2/3}\left(1+\frac{\Delta n}{n_0}\right)^{2/3} = An_0^{2/3}\left(1+\frac{2}{3}\frac{\Delta n}{n_0}\right)$$

so that

$$\Delta x = x(\Delta n) - x_0 = \frac{2}{3}\, x_0 \frac{\Delta n}{n_0}$$

The probability of a deviation Δn from the degree of polymerization n is determined by Bolzmann's function

$$W(\Delta n) = W_0 e^{-\frac{u_0 \Delta n}{kT}}$$

The probability of a deviation from the average sedimentation boundary x_0 by the magnitude Δx corresponds to

$$\frac{\Delta n}{n_0} = \frac{3\Delta x}{2x_0}$$

hence

$$W^*(\Delta x) = We^{-\frac{3u_0}{2kT}\frac{\Delta x n_0}{x_0}}$$

Let us now find the mean quadratic deviation

$$\frac{\sqrt{\overline{\Delta x^2}}}{x_0} = \frac{2}{3}\frac{\sqrt{\overline{\Delta n^2}}}{n_0}$$

Now, as has been shown above, $\sqrt{\overline{\Delta n^2}} = 3 \cdot 0.8$; hence the relative extension of the sedimentation boundary, owing to the statistical spreading of the molecular weights, is finally found to be:

$$\frac{\sqrt{\overline{\Delta x^2}}}{x_0} = \frac{2 \cdot 3 \cdot 0.4}{3 \cdot 150} \cong 0.5 \text{ per cent}$$

The spreading due to the diffusion on the sedimentation diagrams of Svedberg reaches $\frac{\Delta x}{x_0} \cong 10$ per cent. The spreading of the boundary introduced by us is superimposed on that due to diffusion and makes about 5 per cent. Hence it is easy to estimate the error introduced by this circumstance in the calculation of the diffusion coefficient D. As is well known, in the case of diffusion $\overline{\Delta x^2} = 2D\tau$, consequently an error of 5 per cent in the value of spreading corresponds to an error of 10 per cent in the value of the diffusion coefficient D.

The statistical spreading of the molecular weights assumed by us is thus in full agreement with the results of the measurements made with the help of the ultracentrifuge.

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EXPERIMENTAL MORPHOLOGY

INVESTIGATION OF WAYS OF FORMATION OF REGENERATION BLASTEMA BASED ON CALCULATION OF MITOTIC COEFFICIENT

By L. W. POLEŽAJEW and G. I. GINZBURG

(Communicated by L. A. Orbeli, Member of the Academy, 20.XI.1943)

For a long time only one theory existed, according to which the regeneration blastema arises on the amputation wound surface of an organ as a result of an intensive reproduction of cells, stimulated by an excessive formation of the products of decomposition of tissues, *i. e.* by the wound hormones, mitogenetic rays and other agents ([2], [12]). However, facts were discovered which necessitate a revision of this conception.

1. Poležajew showed that in the case of a heterotopic transplantation, young regeneration blastema of the tail ([5],[7]) and limb ([8]) in axolotls are incapable of growth but after an increase in the amount of the material of their subepithelial part they grow well, forming tail-like organs and limbs, respectively. It is thus evident that up to a certain stage the growth of a blastema depends on the accumulation in it of cellular material; this accumulation seems to take place at the expense of an influx of cells without proliferation rather than at the expense of proliferation.

2. Kazanzev ([4]) observed that in the course of a histological investigation of regenerating limbs mitoses were found in relatively late stages of regeneration.

3. It was found that in the process of regeneration of limbs in amphibiae an important part is played by the dedifferentiation of tissues of mesodermal origin leading to a rapid accumulation of cells in the region of the amputation wound surface ([6],[9–11]).

The above facts lead to the conclusion that «the question about the mode of formation of the regeneration bud should be solved experimentally, by calculating the mitotic coefficient at successive stages of the initial phases of the regeneration blastema» ([6]). Special investigations ([9–11]) showed that two successive phases arise in the course of the formation of a regeneration blastema: 1) primary accumulation of cells, mainly without mitoses; 2) secondary accumulation of cells, mainly due to mitotic cell division. Recently such an investigation was carried out also by Bassina ([1]). Her data go fully to confirm our earlier data and conclusions, but her inference that «the accumulation of regeneration cellular material of the blastema occurs at the early stage of its development at the expense of the influx of cells from the proximal part of the limb and at the expense of amitotic division», is inconvincing.

The question concerning the mode of formation of the regeneration blastema is of great importance for the understanding of the mechanism of the regeneration process. In this connexion we undertook another investigation of the question. The experiment was carried out on axolotls one and a half year old. During the period from the 1st to the 23rd of August, 1939, regeneration was studied of simultaneously amputated anterior limbs until their complete restoration. Regeneration proceeded

rapidly, and by the 23rd day the limb already possessed clearly expressed rudiments of digits. The regenerated limbs were daily amputated. The mitotic coefficient (MC), *i. e.* ratio of the number of mitoses to the total number of dividing and non-dividing cells, was calculated for the regenerates at the age of 4, 6, 8, 10, 12, 14, 16, 18 and 22 days. Where it was not clear whether mitosis is beginning, prophases were not counted. Only data concerning the subepithelial part of the regeneration blastema are given here.

The following data were obtained (in pro mille):

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conclusion speak the data on the rôle of the distal part of the organ in regeneration ([12], [3]) and the data on the importance of the profound dedifferentiation of tissues observed during regeneration ([6—7], [9—11]).

The data on the existence of two phases in the process of accumulation of cells in the regeneration blastema (primary and secondary accumulation) lead us to a necessity of reconsidering the question concerning the mechanism of the regeneration process. The mutilation of an organ is of importance not so much as an agent causing proliferation at the first phase of regeneration, but chiefly as an agent whose effect is to establish new relations between tissues (the epithelium and the tissues of mesodermal origin in the remains of the organ), and to cause a profound dedifferentiation of the tissues. The growth of the blastema at the expense of proliferation seems also to be due not so much to the cells being acted upon by the products of tissue decomposition, as to the accumulation of a definite minimum of cells in the blastema and to the formation of some new structure in it.

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CONTRAST SENSITIVITY OF CENTRAL VISION AS AFFECTED BY PRELIMINARY LIGHT STIMULATION OF DIFFERENT AREAS OF RETINA

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From a number of studies it is known that light stimuli are liable to bring about an increase of photo-sensitivity of the peripheral vision ([1—9]), as well as of the electric sensitivity of the eye ([10—12]).

The aim of the experiments here described was to find out the way in which preliminary light stimuli («additional lighting») may affect a visual function of such a high practical importance as the contrast sensitivity of central vision.

Method. The contrast sensitivity of central vision was determined by measuring the minimum illumination at which the observer was found able to detect the object (of a visual size of 13 and 30 minutes) against a darker background (the contrast being equal to 400 and 125 per cent, respectively). Such measurements of threshold illumination were performed during a two-hour long dark adaptation.

The observers were kept in the dark during 45—50 min. after which their eyes were subjected to illumination («additional lighting»), the effect of which was studied from the subsequent run of the dark-adaptation curve. The experiments involved 21 observers, and a total of about hundred adaptation curves have been obtained. The additional lighting applied affected either mainly the scotopic apparatus of the eye (brightness about 0.1 lx against a white background), or the photopic apparatus (illumination with red light which did not affect the rods; brightness about 4 lx against a white background).

There were also tests in which both the scotopic and the photopic apparatus were affected (white light, brightness of about 4 lx against a white background). The additional lighting affected either the centre of the retina (a disc with a diameter of 4°), or its periphery (a ring extending from the 5th to the 20th degree), either both the centre and the periphery (the «whole» field of vision). The length of exposure was 2 minutes for all main experimental sets.

The effect of such additional lighting upon the contrast sensitivity of the central scotopic and photopic vision was investigated. In the former case we determined the contrast sensitivity of the parafoveal regions of the retina to white objects of very low brightness (of the order of 10^{-4} lx against a white background).

In the latter case the object to be discerned was illuminated with red light (brightness of the order of hundredth of a lux against a white background).

When recording the results, as an index of the level of the contrast sensitivity of the eye there was taken the reciprocal of the threshold illumination obtained.

Results and Conclusions. The results of all experiments are summarized in the table. The figures denote the reciprocals of the threshold illu-

mination, and are the medians from all values recorded for different observers of a given experimental set.

The respective value, corresponding to the 50th minute of dark adaptation, has in all cases been put at 100.

	Observer's stay in the dark after additional lighting, in minutes							
	1.5	7	15	25	35	45	55	65
Exper. series I[1]	99	78	88	76	100	—	—	—
» » II[2]	131	78	103	168	149	166	78	83
» » III[3]	165	133	167	190	210	175	—	—
» » IV[4]	87	65	82	90	105	—	142	—
» » V[5]	82	47	76	80	75	—	—	—
» » VI[6]	90	129	107	99	92	82	—	—
» » VII[7]	110	128	171	149	143	137	—	—
» » VIII[8]	81	92	93	103	100	100	—	—
» » IX[9]	57	135	182	235	321	285	137	—
» » X[10]	55	98	84	100	96	100	100	—
» » XI[11]	100	230	223	260	238	245	185	—

[1] Scotopic apparatus of periphery affected by additional lighting; contrast sensitivity of macular rods measured.

[2] Scotopic apparatus of macular region affected by additional lighting; contrast sensitivity of macular rods measured.

[3] Photopic apparatus of periphery affected by additional lighting; contrast sensitivity of macular rods measured.

[4] Photopic apparatus of retinal centre affected by additional lighting; contrast sensitivity of macular rods measured.

[5] Photopic apparatus of periphery affected by additional lighting; contrast sensitivity of macular cones measured.

[6] Photopic apparatus of macular region affected by additional lighting; contrast sensitivity of macular cones measured.

[7] Scotopic apparatus of periphery affected by additional lighting; contrast sensitivity of macular cones measured.

[8] Scotopic apparatus of centre affected by additional lighting; contrast sensitivity of macular cones measured.

[9] Both cones and rods of centre and periphery affected by additional lighting; contrast sentitivity of macular rods measured.

[10] Both cones and rods of retinal centre affected by additional lighting; contrast sensitivity of macular rods measured.

[11] Both cones and rods of retinal periphery affected by additional lighting; contrast sensitivity of macular rods measured.

As may be seen from the table, all the additional lightings applied by us produced as their after-effect rather appreciable and stable alterations of the level of the contrast sensitivity of the eye. This effect was found to vary according to the visual apparatus affected by additional lighting and to the visual apparatus tested upon exposure.

The picture offered by the alterations of sensitivity which set in after additional lighting, may be interpreted as a result of the phenomena of direct sensibilization ([3],[8]), reciprocal inhibition and subsequent desinhibition connected with increased excitability ([3], [13-15]) which are developing in the visual apparatus.

When applied to practice, in order to raise in the course of dark adaptation the contrast sensitivity of central photopic vision, the data obtained authorize us to recommend the use of a short (1 — 2 minute long) exposure to light of low intensity, affecting only the scotopic apparatus of the peripheral regions of the retina. In order to heighten the contrast sensitivity of macular scotopic vision, it is found very useful to apply 1—2 minute long stimuli (brightness of about 4 lx against a white background) to the photopic apparatus of more peripheral regions of the retina.

On the contrary, in the case of central scotopic vision the subsequent contrast sensitivity of central vision is found to be reduced appreciably by additional lighting affecting the peripheral rods; for the central photopic vision, it is the illumination affecting the peripheral photopic apparatus which produces a similar effect.

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